Dividing Polynomials; Remainder and Factor Theorems

Section 2.4

Objectives
– Use long division to divide polynomials.
– Use synthetic division to divide polynomials.
– Evaluate a polynomial using the Remainder Theorem.
– Use the Factor Theorem to solve a polynomial equation.

Warm Up!
• What are the factors of 12?
• Why is 5 not a factor?
• How do you define a factor?

This leads us to….
DIVISION!!

How do you divide a polynomial by another polynomial?
• Perform long division, as you do with numbers! Remember, division is repeated subtraction, so each time you have a new term, you must subtract it from the previous term.
• Work from left to right, starting with the highest degree term.
• Just as with numbers, there may be a remainder left. The divisor may not go into the dividend evenly.

Example
• Divide using long division. State the quotient, q(x), and the remainder, r(x).
  \[(6x^3 + 17x^2 + 27x + 20) \div (3x + 4)\]
Example
• Divide using long division. State the quotient, \( q(x) \), and the remainder, \( r(x) \).
\[
\left( 4x^3 - 8x + 6 \right) \div (2x - 1)
\]

Remainders can be useful!
• The Remainder Theorem states:
  If the polynomial \( f(x) \) is divided by \( (x - c) \), then the remainder is \( f(c) \).
• If you can quickly divide, this provides a nice alternative to evaluating \( f(c) \).

Synthetic Division
• Quick method of dividing polynomials
• Used when the divisor is of the form \( x - c \)
• Last column is always the remainder

Example
• Divide using synthetic division.
\[
\frac{x^3 + x - 2}{x - 1}
\]

Example
• Divide using synthetic division.
\[
\frac{x^3 - 2x^4 - x^3 + 3x^2 - x + 1}{x - 2}
\]

So, again...
The Remainder Theorem
If the polynomial \( f(x) \) is divided by \( x - c \), then the remainder is \( f(c) \).
If you are given the function \( f(x) = x^3 - 4x^2 + 5x + 3 \) and you want to find \( f(2) \),
then the remainder of this function when divided by \( x - 2 \) will give you \( f(2) \)

\[
\begin{array}{c|cccc}
  2 & 1 & -4 & 5 & 3 \\
  \hline
  & 2 & -4 & 2 \\
  & 1 & -2 & 1 \\
  \rightarrow & & & 5 \\
\end{array}
\]

\( f(2) = 5 \)
Example
• Use synthetic division and the Remainder Theorem to find the indicated function value.
  \[ f(x) = x^3 - 7x^2 + 5x - 6; \ f(3) \]

\[ f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6; \ f(2) \]

Now, let’s talk about a new theorem.

The Factor Theorem
Let \( f(x) \) be a polynomial.
\[ \text{a. If } f(c) = 0, \text{ then } x - c \text{ is a factor of } f(x). \]
\[ \text{b. If } x - c \text{ is a factor of } f(x), \text{ then } f(c) = 0. \]

What does it mean if \( f(c) = 0 \)?
(Hint: Think about where this point would be located on the coordinate plane.)

Factor Theorem
– If \( f(c) = 0 \), then we have the point \((c,0)\).
– This point is on the x-axis…which represents an x-intercept…which is also called a ZERO!!!

SO……
• If we know a factor, we know a zero!
• If we know a zero, we know a factor!

Zero of Polynomials
If \( f(x) \) is a polynomial and if \( c \) is a number such that \( f(c) = 0 \), then we say that \( c \) is a zero of \( f(x) \).
The following are equivalent ways of saying the same thing.
1. \( c \) is a zero of \( f(x) \)
2. \( x - c \) is a factor of \( f(x) \)

Example
Solve the equation \( 2x^3 - 3x^2 - 11x + 6 = 0 \) given that 3 is a zero of \( f(x) = 2x^3 - 3x^2 - 11x + 6 \).

Divide \( \left( x^3 + x^2 + 2x - 8 \right) \div \left( x - 3 \right) \)
(a) \( x^3 + x - 8 \)
(b) \( x^3 - 4x + 2 \)
(c) \( x^3 + 4x + 14 \)
(d) \( x^3 + 4x + 14 + \frac{24}{x - 3} \)
Use Synthetic Division and the Remainder Theorem to find the value of \( f(2) \) for the function
\[ f(x) = x^3 + x^2 - 11x + 10 \]
(a) 2
(b) 0
(c) −5
(d) −12