Exponential Growth & Decay

Objective: To apply models of exponential growth and decay.

Could the following graph model exponential growth or decay?

- a) Growth model.
- b) Decay model.

Exponential Growth & Decay Models

- \( A = A_0 e^{kt} \)
  - \( A_0 \) is the amount you start with, \( t \) = time, and \( k \) = constant of growth (or decay)
  - If \( k > 0 \), the amount is GROWING (getting larger), as in the money in a savings account that is having interest compounded over time
  - If \( k < 0 \), the amount is SHRINKING (getting smaller), as in the amount of radioactive substance remaining after the substance decays over time

Example – Watch and listen.

Population Growth of the United States.

In 1990 the population in the United States was about 249 million and the exponential growth rate was 8% per decade. (Source: U.S. Census Bureau)
- Find the exponential growth function.
- What will the population be in 2020?
- After how long will the population be double what it was in 1990?

Solution

- At \( t = 0 \) (1990), the population was about 249 million. We substitute 249 for \( P_0 \) and 0.08 for \( k \) to obtain the exponential growth function.
  \[ P(t) = 249e^{0.08t} \]
- In 2020, 3 decades later, \( t = 3 \). To find the population in 2020 we substitute 3 for \( t \):
  \[ P(3) = 249e^{0.08(3)} = 249e^{0.24} = 317. \]
  The population will be approximately 317 million in 2020.

Solution continued

- We are looking for the doubling time \( T \).
  \[ 2 = 249e^{0.08T} \]
  \[ \ln 2 = \ln e^{0.08T} \] (Taking the natural logarithm on both sides)
  \[ \ln 2 = 0.08T \] (\( \ln e^x = x \))
  \[ \frac{\ln 2}{0.08} = T \]
  \[ 8.7 = T \]

The population of the U.S. will double in about 8.7 decades or 87 years. This will be approximately in 2077.
Example
- In 1990, the population of Africa was 643 million and by 2000 it had grown to 813 million.
- a) Use the exponential growth model $A = A_0 e^{kt}$ in which $t$ is number of years after 1990, to find the exponential growth function that models the data.

Example continued:
- b) By which year will Africa’s population reach 2000 million, or two billion?

Example
- Strontium-90 is a waste product from nuclear reactors. As a consequence of fallout from atmospheric nuclear tests, we all have a measurable amount of strontium-90 in our bones.
- a) The half-life of strontium-90 is 28 years, meaning that after 28 years a given amount of the substance will have decayed to half the original amount. Find the exponential decay model for strontium-90.

Example continued
- b) Suppose that a nuclear accident occurs and releases 60 grams of strontium-90 into the atmosphere. How long will it take for strontium-90 to decay to a level of 10 grams?

Help for Test 3!
- Be sure to see the Graphic Organizer “Logarithm & Exponential Summary” on my website for a summary of Test 3 material.