Exponential & Logarithmic Equations

Objective: To solve exponential and logarithmic equations.

Why it’s important...

This skill can be used to solve many types of real-world problems, including those involving animal population.

Definition

Exponential equation – form $b^x = a$; (an equation with a variable in the exponent)

Example: $7^{3x} = 20$

Solving with Same Base

If $b^M = b^N$, then $M = N$.

STEPS
1. Re-write the equation in the form $b^M = b^N$.
2. Set $M = N$.
3. Solve for the variable.

Check Point 1: Solve

a) $5^{3x - 6} = 125$  

b) $8^{x + 2} = 4^{x - 3}$

Key Question

How do we solve an exponential equation in which we can’t re-write with the same base?

Answer: Take the natural or common logarithm of each side and solve.
Using Logs to Solve

1. Isolate the exponential expression.
2. Take the common or natural logarithm of both sides.
3. Simplify using properties. (Remember: \( \log b^x = x \log b \) or \( \ln e^x = x \).)
4. Solve for the variable.

See Example 2, page 411.

Check Point 2: Solve \( 5^x = 134 \).

See Example 3, page 412.

Check Point 3: Solve \( 7 e^{2x} - 5 = 58 \).

Watch and listen.

Solve: \( 2^x = 50 \)

\[
\log 2^x = \log 50
\]

\[x \log 2 = \log 50\]

\[x = \frac{\log 50}{\log 2}\]

This is an exact answer. We cannot simplify further, but we can approximate using a calculator.

\( x \approx 5.6439 \)

We can check by finding \( 2^{5.6439} \approx 50 \).

Why can’t we simplify \( 50/2 \) to \( 25 \)?

Find each value below and compare.

\[
\frac{\log 50}{\log 2} \quad \log 25
\]

See Example 4, page 412.

Check Point 4: Solve \( 3^{2x - 1} = 7^x + 1 \).
Check Point 5: Solve $e^{2x} - 8e^x + 7 = 0$.

More Practice – Solve.

A) $3^x = 4$  
B) $6^{2x} = 21$

More Examples

C) $3^{x+4} = 101$  
D) $e^{-0.25w} = 12$

Solving Logarithmic Equations

Equations containing variables in logarithmic expressions, such as $\log_2 x = 16$ and $\log x + \log (x + 4) = 1$, are called logarithmic equations.

To solve logarithmic equations

1. Obtain a single logarithmic expression on one side.
2. Write an equivalent exponential equation.
3. Solve and check.

See Example 6, page 413-414.

Check Point 6
a) $\log_2 (x - 4) = 3$  
b) $4 \ln(3x) = 8$
Check Point 7
Solve: \( \log x + \log (x - 3) = 1 \).

More Examples
\[ \text{Solve: } \log_2 x = -3 \]

Another Example
\[ \text{Solve: } \log_2 (x + 1) + \log_2 (x - 1) = 3 \]

Check Solution to Example
\[ \text{Check: For } x = 3: \log_2 (3 + 1) + \log_2 (3 - 1) = 3 \rightarrow \]
\[ \log_2 4 + \log_2 2 = 3 \]
\[ \log_2 (4 \cdot 2) = 3 \]
\[ \log_2 8 = 3 \]
\[ 3 = 3 \]

\[ \text{For } x = -3: \log_2 (-3 + 1) + \log_2 (-3 - 1) = 3 \]
\[ \log_2 (-2) + \log_2 (-4) = 3 \]

Negative numbers do not have real-number logarithms. The solution is 3.

Using the One-to-One Property
\[ \text{Get a single log on each side.} \]
\[ \text{Use the property:} \]
\[ \text{If } \log_b M = \log_b N, \text{ then } M = N. \]
\[ \text{Solve for the variable.} \]

See Example 8, page 415
\[ \text{Check Point 8} \]
Solve \( \ln (x - 3) = \ln (7x - 23) - \ln (x + 1) \).
Extra Example

Solve: \( \ln x - \ln(x - 4) = \ln 3 \)

Applications – See Example 9.

Check Point 9: Use the formula in example 9 to answer the question: “What blood alcohol concentration corresponds to a 7% risk of a car accident? (In many states, drivers under the age of 21 can lose their licenses for driving at this level.)

Applications – See Example 10.

Check Point 10: How long, to the nearest tenth of a year, will it take $1000 to grow to $3600 at 8% annual interest compounded quarterly?

Applications – See Example 11.

Check Point 11: Use the function in Example 11 to find in which year there will be 210 million Internet users in the United States.

Additional Examples

a) \( 7^{5x} = 3000 \)

b) \( 5^{2x} = 16 \)

c) \( \log(7 - 2x) = -1 \)
Additional Examples

d) \( \log 6 - \log 3x = -2 \)