Objectives

– Solve polynomial inequalities.
– Solve rational inequalities.

Polynomial and Rational Inequalities

Section 2.7

Graph \( P(x) = x^3 + x^2 - 6x \) on a graphing calculator and solve the following.

1.) \( P(x) = 0 \)
2.) \( P(x) > 0 \)
3.) \( P(x) < 0 \)
4.) \( P(x) \geq 0 \)
5.) \( P(x) \leq 0 \)

Given the following graph of \( f(x) \), give interval notation for \( x \)-values such that \( f(x) > 0 \).

1) \((-3,-1) \cup (0,2) \cup (4,\infty)\)
2) \((-\infty,-3) \cup (-1,0) \cup (2,4)\)
3) \((-\infty,\infty)\)
4) \((-3,4)\)

Solving Polynomial Inequalities

- Always compare the polynomial to zero.
- Factor the polynomial. We are interested in when factors are either positive or negative, so we must know when the factor equals zero.
- The values of \( x \) for which the factors equal zero provide the cut-offs for regions to check if the polynomial is positive or negative.
Procedure for Solving Polynomial Inequalities

1. Express the inequality in the form \( f(x) > 0 \) or \( f(x) < 0 \) where \( f \) is a polynomial function.

2. Solve the equation \( f(x) = 0 \). The real solutions are the boundary points.

3. Locate these boundary points on a number line, dividing the number line into intervals.

4. Choose one representative number, called a test value, within each interval and evaluate \( f \) at that number.
   
   a.) If the value of \( f \) is positive, then \( f(x) > 0 \) for all numbers, \( x \), in the interval.

   b.) If the value of \( f \) is negative, then \( f(x) < 0 \) for all numbers, \( x \), in the interval.

5. Write the solution set, selecting the interval or intervals that satisfy the given inequality.

This procedure is valid if \(<\) is replaced by \(\leq\) or \(>\) is replaced by \(\geq\). However, if the inequality involves \(<\) or \(>\), include the boundary points in the solution set.

Example

• Solve \( 3x^2 + 16x < -5 \)

Solving Rational Inequalities

• VERY similar to solving polynomial inequalities EXCEPT if the denominator equals zero, there is a domain restriction. The function COULD change signs on either side of that point.

• Step 1: Compare inequality to zero. (Add constant to both sides and use a common denominator to have a rational expression.)

• Step 2: Factor both numerator & denominator to find “cut-off” values for regions to check when function becomes positive or negative.

Example

• Solve \( \frac{x + 3}{x^2 - 1} \geq 0 \)