Sets

Sets and Elements

- A set is any collection of objects. The objects, which may be countries, cities, years, numbers, letters, or anything else, are called the elements of the set.
- A set is often specified by a listing of its elements inside a pair of braces. A set may also be specified by giving a description of its elements.

Example

1. The set of the first six letters of the alphabet is \{a, b, c, d, e, f\}.
2. \{2, 4, 6, 8, 10\} is the set of the even numbers between 1 and 11.
3. The graph \{(a,b)\ where \ b = a^2\} is a set with infinitely many elements.

Union and Intersection

- The union of \(A\) and \(B\), denoted \(A \cup B\), is the set consisting of all those elements that belong to either \(A\) or \(B\) or both.
- The intersection of \(A\) and \(B\), denoted \(A \cap B\), is the set consisting of those elements that belong to both \(A\) and \(B\).

Example

- Let \(A = \{2, 4, 6, 8, 10\}\) and \(B = \{1, 2, 3, 4\}\). Find \(A \cup B\).
- Let \(A = \{\text{red, blue, green, yellow}\}\) and \(B = \{\text{red, blue}\}\). Find \(A \cap B\).

Subset and Empty Set

- A set \(B\) is called a subset of \(A\) provided that every element of \(B\) is also an element of \(A\).
- The set that contains no elements at all is the empty set (or null set) and is written as \(\emptyset\) or \(\{\}\). The empty set is a subset of every set.
Example
- List all possible subsets of \{a, b, c\}.
  - \{
  \}
  - \{a\}, \{b\}, and \{c\}
  - \{a, b\}, \{a, c\}, and \{b, c\}
  - \{a, b, c\}

Total Number of Subsets
- A set of \(n\) elements has \(2^n\) subsets.
- Suppose \(U = \{1, 2, 3, 4, 5, 6, 7\}\).
  Find the number of subsets of \(U\).

Universal Set and Complement
- The set \(U\) that contains all the elements of the sets being discussed is called a **universal set** (for the particular problem).

- If \(A\) is a subset of \(U\), the set of elements in \(U\) that are not in \(A\) is called the **complement** of \(A\), denoted by \(A'\).

Example
Let \(U = \{a, b, c, d, e, f, g\}\), \(S = \{a, b, c\}\) and \(T = \{a, c, d\}\).
Find:

Additional Examples
Let \(U = \{1, 2, 3, 4, 5\}\), \(S = \{1, 2, 3\}\), and \(T = \{5\}\).
List the elements of the following sets.

- a.) \(S'\)
- b.) \(S \cup T\)
- c.) \(S \cap T\)
- d.) \(S' \cap T\)

Additional Examples
Let \(U = \{1, 2, 3, 4, 5\}\), \(R = \{1, 3, 5\}\), \(S = \{3, 4, 5\}\), and \(T = \{2, 4\}\).
List the elements of the following sets.

- a.) \((R \cup S)'\)
- b.) \(R \cap S \cap T\)
- c.) \((R \cup S) \cap (R \cup T)\)
Additional Examples

A large corporation classifies its many divisions by their performance in the preceding year.
Let \( P = \{ \text{divisions that made a profit} \} \),
\( L = \{ \text{divisions that had an increase in labor costs} \} \), and
\( T = \{ \text{divisions whose total revenue increased} \} \).
Describe the sets in the following using set-theoretic notation.

a.) \{ \text{divisions that did not make a profit} \}

b.) \{ \text{divisions that had an increase in labor costs and either were unprofitable or did not increase their total revenue} \}