For the next several lessons we’ll be exploring fractions and operations with fractions and the emphasis will be on understanding the concept of a fraction rather than rules for manipulation.

Research has shown that “procedural knowledge, such as algorithms for operations, is often taught without contexts or concepts, implying to the learner that algorithms are an ungrounded code only mastered through memorization. Introducing algorithms before conceptual understanding is established, or without linking the algorithm to conceptual knowledge, creates a curriculum that tends to be perplexing for children to master or appreciate.”

Memorization without __________ often leads to misapplication of the algorithm.

Concepts must be placed in _______. In most cases, working abstractly with numbers does not foster understanding unless a foundation has been previously laid so that a child can make __________.

Putting concepts into situations with which children are familiar is crucial. Meaningful learning depends on connecting the new concept to the existing knowledge base in some way.

When we say “½” we are implicitly referring to ½ of something.

Every fraction has a “whole” or base of reference associated with it. Contexts can help one to focus on what that “whole” is.

Consider ½ of a 12-inch pizza and ½ of 16-inch pizza. Are they the same? Different? Explain.
WHAT ARE RATIONAL NUMBERS?
(WHAT ARE FRACTIONS?)
Numbers that can be written as a comparison of two integers, a/b, b ≠ 0.
Which of the following are rational numbers?
-3/4, 5, 1/5, 0, 0.37, 1/25

MODELING RATIONAL NUMBERS
• Identifying the Whole and Separating It into Equal Parts (Pictorial Representation)
  2/3: Dividing a whole into equal size parts and identifying two of those parts
• Using Two Integers to Describe Part of a Whole
  3 slices of pizza / 8 slices of pizza (whole pizza)
• Using Fraction Language
  “halves” “thirds” “fourths”

RATIONAL NUMBERS VS. FRACTIONS
• A rational number is the relationship represented by an infinite set of ordered pairs, each of which describes the same quantity.
• A fraction is a symbol, a/b, where a and b are numbers and b ≠ 0. Here, a is the numerator of the fraction and b is the denominator of the fraction.

REPRESENTING AND DESCRIBING FRACTIONS
• Write definitions and draw pictorial representations for the following fractions.
  1) 1/3
  2) 3/5
  3) 5/4

TWO TYPES OF FRACTIONS
• When the numerator of a fraction is less than the denominator, the fraction is called a _______ fraction.
• When the numerator of a fraction is greater than or equal to the denominator, the fraction is called an _______ fraction.

PAPER-FOLDING ACTIVITY
• Take a piece of paper and fold it in half. Label the fractions represented by each rectangle formed.
• Fold the paper in half again. What fractions can be represented now? Label them.
• Fold the paper in half once again. Discuss different fraction interpretations of the rectangles formed. Label again.
• What is the significance of this activity? (What concept is being introduced?)
EQUIVALENT FRACTIONS

Two fractions, \(a/b\) and \(c/d\), are _______ fractions iff \(ad = bc\).

Fundamental Law of Fractions
Given a fraction \(a/b\) and a number \(c \neq 0\), \(a/b = ac/bc\).

SIMPLIFYING FRACTIONS

A fraction representing a rational number is in _______ form when the numerator and denominator are both integers that are relatively prime and the denominator is greater than zero.

EXAMPLES- WRITE EACH IN LOWEST TERMS. (AKA “SIMPLIFY”.)

\[
a) \quad \frac{12}{36} = \\
b) \quad \frac{4}{28} = \\
c) \quad \frac{18}{45} = 
\]

FAIR SHARE ACTIVITY

For each of the following problems, imagine that you have the given number of brownies to share equally among a certain number of people. Find out how many (or how much of a) brownies each person gets.

Explain your process and reasoning. In any stage of the process, if you talk about or use a fraction, be sure to write the expression for the fraction. Be sure to label any diagrams with appropriate fraction notation. Write your answer as a fraction or sum of fractions that expresses your process (not just the final answer).

3 people share 4 brownies
4 people share 7 brownies
4 people share 2 brownies
3 people share 2 brownies

FOUR MEANINGS OF ELEMENTARY FRACTIONS

1) Part of a Whole
2 slices of a pizza cut into 8 equal slices

2) Part of a Group or Set
3/5 of a group of 20 people prefer juice over milk.

3) Position on a Number Line
A scarf 3 1/2 feet long made from a length of silk 5 feet long.

4) Division
1 chocolate cream pie split between four people

Elementary fractions will most likely not deal with rational values represented by negative fractions, nor irrational fractions.

DECIMALS

A __________ is a symbol that uses a base-ten place-value system with tenths and multiples of tenths to represent a number. A decimal point is used to identify the ones place.
WAYS TO EXPRESS DECIMALS

- Expanded notation
- As a fraction

WAYS TO EXPRESS DECIMALS

Examples:
1) Express 31.25 in expanded notation.

CONVERTING FRACTIONS TO DECIMALS

- Using the Fundamental Law of Fractions
  - Multiply the numerator and denominator by some value that will produce a product in the denominator that can be written as a power of 10.

CONVERTING FRACTIONS TO DECIMALS

- Using Division
  - Divide the numerator by the denominator using the standard algorithm for division.

Examples:
2) Write 0.75 and 1.3 as simplified fractions.

Examples:
1) 3/25
2) 1/4
3) 4/5
CONVERTING FRACTIONS TO DECIMALS

Examples: Use division to change each fraction to a decimal.
1) \(\frac{7}{8}\)
2) \(\frac{9}{11}\)

TYPES OF DECIMALS

When using division to change from fractions to decimals, the remainder determines the type of decimal.

- If the remainder finally becomes 0, then the resulting decimal has a fixed number of places and is called a terminating decimal. With a terminating decimal, the denominator can be expressed as a power of ten (using the Fundamental Law of Fractions).
- If the remainder will never become 0, then the decimal in the quotient has a digit or group of digits that will repeat over and over. This is called a repeating decimal.

Thus, every rational number can be expressed as terminating or repeating decimal.

SCIENTIFIC NOTATION

A rational number is expressed in scientific notation when it is written as a product where one factor is a greater than or equal to 1 and less than 10 and the other factor is a product of 10.

SCIENTIFIC NOTATION EXAMPLE

Write using scientific notation.

A) 275 billion
B) 0.000000457