Inverse Functions

Objectives
- Verify inverse functions
- Find the inverse of a function.
- Use the horizontal line test to determine one-to-one.
- Given a graph, graph the inverse.
- Find the inverse of a function & graph both functions simultaneously.

What is an inverse function?
- A function that “undoes” the _______ function.
- A function “wraps an x” and the inverse would “unwrap the x” resulting in x when the 2 functions are composed on each other.
- The domain of f is equal to the _______ of $f^{-1}$ and vice versa.

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

How do their graphs compare?
- The graph of a function and its inverse always _______ each other through the line $y = x$.
- Example: $y = \frac{1}{3}x + 2$ and its inverse $= 3(x - 2)$
- Every point on the graph $(x,y)$ exists on the inverse as $(y,x)$ (i.e. if (-6,0) is on the graph, (0,-6) is on its inverse.

Example 1
- Show that each function is the inverse of the other.
(Verify by showing $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.)
- $f(x) = 4x - 7$ and $g(x) = \frac{x + 7}{4}$

How do you find an inverse?
1. Replace $f(x)$ with $y$.
2. Interchange (__________) the $x$ and the $y$ in the equation.
3. Solve for _______.
4. If $f$ has an inverse, replace $y$ in step 3 with $f^{-1}(x)$.
5. Verify.
**Example 3**

- Find the inverse of \( f(x) = 2x + 7 \).

**Example 4**

- Find the inverse of \( f(x) = 4x^3 - 1 \).

**Example 5**

Find the inverse of \( f(x) = \frac{3}{x} - 1 \).

**Do all functions have inverses?**

- Yes, and no. Yes, they all will have inverses, BUT we are only interested in the inverses if they ARE A __________.

**DO ALL FUNCTIONS HAVE INVERSES THAT ARE FUNCTIONS? NO.**

- Recall, functions must pass the vertical line test when graphed. If the inverse is to pass the vertical line test, the original function must pass the __________ line test (be one-to-one)!

**Horizontal Line Test**

- If a horizontal line intersects a graph in only one point, then the function is one-to-one and has an inverse that is a function.

**Graphing the Inverse**

- Remember: The inverse of a function is a reflection about the line \( y = x \).
- To graph, simply swap each \( x \) & \( y \).
Example 6

- The graph of function $f$ consists of two line segments, one segment from (-2, -2) to (-1, 0) and a second segment from (-1, 0) to (1,3). Graph $f$ and use the graph to draw the graph of its inverse function.