REVIEW
Reminder: Domain Restrictions

For FRACTIONS:
- No zero in denominator!
  ex. $\frac{7}{0}$ = undefined

For EVEN ROOTS:
- No negative under radical!
  ex. $\sqrt{x - 2}$, $\sqrt[4]{x - 2}$

Review: Find the domain of each and write in interval notation.

a) $f(x) = |3x - 2|$

b) $f(x) = \frac{1}{x + 4}$

c) $g(x) = \sqrt{x - 7}$

d) $f(x) = \sqrt{7 - x}$

More on Functions

Objectives
- To find the difference quotient.
- Understand and use piecewise functions.
- Identify intervals on which a function increases, decreases, or is constant.
- Use graphs to locate relative maxima or minima.
- Identify even or odd functions & recognize the symmetries.
- Graph step functions.

Functions & Difference Quotients

- Useful in discussing the rate of change of function over a period of time.
- EXTREMELY important in calculus (h represents the difference in two x values)

DIFFERENCE QUOTIENT FORMULA:

\[
\frac{f(x + h) - f(x)}{h}
\]
Difference Quotient
The average rate of change (the slope of the secant line)

If $f(x) = -2x^2 + x + 5$, find and simplify

A) $f(x + h)$

If $f(x) = -2x^2 + x + 5$, find and simplify

B) $\frac{f(x + h) - f(x)}{h}$

c) Your turn: Find the difference quotient: $f(x) = 2x^2 - 2x + 1$

PIECEWISE FUNCTIONS

- Piecewise function – A function that is defined differently for different parts of the domain; a function composed of different “pieces”

Note: Each piece is like a separate function with its own domain values.

Examples: You are paid $10/hr for work up to 40 hrs/wk and then time and a half for overtime.

$f(x) = \begin{cases} 
10x, & x \leq 40 \\
10(40) + 15(x - 40), & x > 40 
\end{cases}$

Example – Cell Phone Plan
($20 for \leq 1 hour plus 40 cents per minute over 60$)

- Use the function $C(t) = \begin{cases} 
20 & 0 \leq t \leq 60 \\
20 + 0.40(t - 60) & t > 60 
\end{cases}$

to find and interpret each of the following:

d) $C(40)$
e) $C(60)$
Graphing Piecewise Functions

- Graph each "piece" on the same coordinate plane.

Functions Defined Piecewise

Graph the function defined as:

\[
  f(x) = \begin{cases} 
  -3 & \text{for } x \leq 0 \\
  -3 + x^2 & \text{for } 0 < x \leq 2 \\
  \frac{x}{2} - 1 & \text{for } x > 2 
  \end{cases}
\]

Piecewise Graphs Extra Example

\[
  f(x) = \begin{cases} 
  4 & \text{for } x \leq -2 \\
  x + 1 & \text{for } -2 < x < 3 \\
  -x & \text{for } x \geq 3 
  \end{cases}
\]

Describing the Function

- A function is described by **intervals**, using its domain, in terms of x-values.
- Remember: \( \infty \) refers to "positive infinity"
- Remember: \( -\infty \) refers to "negative infinity"

Increasing and Decreasing Functions

- **Describe by observing the x-values.**

  - **Increasing:** Graph goes "up" as you move from left to right.
    \( x_1 < x_2, f(x_1) < f(x_2) \)

  - **Decreasing:** Graph goes "down" as you move from left to right.
    \( x_1 < x_2, f(x_1) > f(x_2) \)

  - **Constant:** Graph remains **horizontal** as you move from left to right.
    \( x_1 < x_2, f(x_1) = f(x_2) \)
Increasing and Decreasing

\[ f(x) = 5 - (x + 1)^2 \]

Increasing \(-\infty, -1\) \(\cup\) \((-1, \infty)\)
Decreasing \((-1, 2)\)

Increasing and Decreasing

\[ f(x) = |x - 2| \]

Decreasing \((-\infty, 2)\)
Increasing \((2, \infty)\)

Constant

\[ g(x) = 4 \]

Constant

Find the Intervals on the Domain in which the Function is Increasing, Decreasing, and/or Constant

Relative Maxima and Relative Minima

- based on “y” values
- maximum – “peak” or highest value
- minimum – “valley” or lowest value

We say, “It has a relative maximum at \((x-value)\) and the maximum is \((y-value)\).”

Relative Maxima and Relative Minima
Even & Odd Functions & Symmetry

- Even functions are those that are **mirrored** through the y-axis. (If \( -x \) replaces \( x \), the y value remains the same.) (i.e. 1st quadrant reflects into the 2nd quadrant)

- Odd functions are those that are mirrored through the **origin**. (If \( -x \) replaces \( x \), the y value becomes \( -y \).) (i.e. 1st quadrant reflects into the 3rd quadrant or over the origin)

Example

- Determine whether each function is even, odd, or neither.
  
  f) \( f(x) = x^2 + 6 \)  
  g) \( g(x) = 7x^3 - x \)

Determine whether each function is even, odd, or neither.

h) \( h(x) = x^5 + 1 \)

i) Your turn: Determine if the function is even, odd, or neither.

\[ f'(x) = 2(x - 4)^2 - 2x^2 \]

- Even
- Odd
- Neither