Solving Radical Equations

Objective: To solve radical equations using the power rule.

Real-World Connection

- You can use radical equations to solve problems involving oceanography and recreation such as designing an amusement park.

DEFINITION

- **Radical equation** – an equation that has a _______ in the radicand
- Example: $3 + \sqrt{x - 4} = 12$

Connection

- We solve radical equations the _______ way we’ve solved earlier equations: isolate the variable, solve and check.
- **Refresher:** Solve $3x + 4 = 2x - 6$.
  
  $3x - 2x = -6 - 4$
  
  $x = -10$

  Check: $3(-10) + 4 = 2(-10) - 6$
  
  $-30 + 4 = -20 - 6$
  
  $-26 = -26$

Power Rule for Solving Radical Equations

- If both sides of an equation are raised to the same power, all solutions of the original equation are also solutions of the new equation.
- *In other words, to “get rid of a radical”, raise _______ sides to the same _______ as the index.*

Steps for Solving Radical Equations

1. Isolate the _______. (Get it on one side of the equation by itself.)
2. Use the power rule to _______ the radical. (Raise both sides to the “root’s” power.)
3. Solve.
4. Check. (Very important!)
Getting Started

- Solve each equation. Check your solution.
  
  \( a) \sqrt{a-4} = 5 \quad b) \sqrt{x+7} = 12 \)

You try...

- \( c) \sqrt{d-2} = 6 \)

Definition

- **Extraneous solution** – a solution that does not satisfy the equation

Note: Extraneous solutions can occur when you square both sides of an equation to create a new equation. Always check your answers to be sure none are “extraneous”.

- **Special note**: If all solutions are extraneous, the equation has “no solution”.

Example

- \( d) \sqrt{5x-3} + 2 = 0 \quad e) 9 - \sqrt{4k+1} = 0 \)

More Examples

- \( f) \sqrt{3t+4} = \sqrt{5t-6} \quad g) \sqrt[5]{m} = \sqrt[5]{5m-16} \)

Even More...

- \( h) 5\sqrt{4x+1} = 3\sqrt{10x+25} \quad i) p = \sqrt{p^2 - 3p + 18} \)