Multiplying & Dividing Radicals

Objectives: To simplify products & quotients containing radicals.

REVIEW
- When adding or subtracting radicals, you must have the _________ radicand.
- Simplify, if needed.
- Add/subtract coefficients and keep the radicand.
Examples:
\[3\sqrt{x} + 4\sqrt{y} - 5\sqrt{y} + 6\sqrt{x}\]
\[\sqrt{50} - 5\sqrt{75}\]

Connection
- Do terms have to be “like” to multiply or divide? Think about using the Distributive Property...for example, to simplify \(3(x + 5)\), do the terms in ( ) have to be like terms?

Multiplying Radicals
- You can multiply or divide any radicals with the same index...they don't have to have “like” radicands.

Multiplying Radicals

STEPS
1. Multiply the _________.
2. Multiply the _________.

Example
\[\text{a) } \sqrt{5}(1 - 3\sqrt{6}) \quad \text{b) } (2\sqrt{6} + 3\sqrt{3})(\sqrt{6} - 5\sqrt{3}) \]
\[\text{c) } (\sqrt{7} + 4)^2\]
Rationalizing the Denominator

- Remember: We should not leave a ________ in the denominator.
- To “rationalize”:
  1. Simplify.
  2. Multiply both parts of the fraction by something that will allow you to take the ________ in the denominator.

Examples

1. \( \frac{11}{\sqrt{11}} \)
2. \( \frac{\sqrt{7}}{\sqrt{6}} \)
3. \( \frac{-\sqrt{13}}{\sqrt{m}} \)

Rationalizing with Cube Roots

1. \( \sqrt[3]{\frac{5}{16}} \)
2. \( -\sqrt[3]{\frac{6x}{y^2}} \)

Definition

- Conjugates -- the sum and difference of the same two terms
- Example: \((3 + \sqrt{x})(3 - \sqrt{x})\)
- Product of Conjugates = DOTS; product has no radical

Find each conjugate & multiply.

\((5 + \sqrt{6})\)
\((x - \sqrt{7})\)
\((\sqrt{2y} - \sqrt{2})\)
Example

\[
a) \frac{4}{\sqrt{7} + \sqrt{5}} \\
b) \frac{-4}{\sqrt{10} + \sqrt{8}} \\
c) \frac{-5}{\sqrt{11} - \sqrt{3}}
\]

Writing Radical Quotients in Lowest Terms

- Simplify radicals.
- Factor.
- Reduce.

Example

\[
a) \frac{12 - 6\sqrt{2}}{24} \\
b) \frac{12 + 9\sqrt{72}}{18} \\
c) \frac{11y + \sqrt{242y^2}}{22y}
\]

SUMMARY

1. Multiply radicals using the Distributive Property or FOIL.
2. In simplified quotients, rationalize the denominator.
3. Multiply by the conjugate to get rid of binomial radicals in the denominator.