Equations with Rational Expressions and Graphs

Objective: To determine the domain of a rational equation and to solve rational equations.

Why is this important?
- You can use rational equations to solve problems involving work, psychology, and electronics.

Determining Domain
- Domain of an equation – all possible values for “x”
- Remember: Restrict the domain values that make the denominator zero!

Determine the domain of each equation.

\[ \frac{3}{x+4} - \frac{2}{x-9} = 0 \]
\[ \frac{6}{4x+7} - \frac{3}{x} = \frac{5}{6x-13} \]
\[ \frac{4x-1}{2x+3} = \frac{12x-25}{6x-2} \]

Steps for Solving
1. Find the LCD. Factor, if necessary.
2. Multiply EVERY part of the problem by the LCD. (Like magic, all fractions will disappear!)
3. Solve.
4. Check. (This is most important!)

Watch. (Pencils down, focus, and listen.)
- Ex. 1) \( \frac{2-r}{4} + \frac{3r}{5} = \frac{r+2}{3} \)
- LCD = 60
- \( (60) \frac{2-r}{4} + (60) \frac{3r}{5} = (60) \frac{r+2}{3} \)
- \( 15(2 - r) + 12(3r) = 20(r + 2) \)
- \( 30 - 15r + 36r = 20r + 40 \)
- \( -15r + 36r - 20r = 40 - 30 \)
- Answer: \( r = 10 \)
To check, substitute answer into original problem.

- Check: \(2 - \frac{10}{4} + \frac{3(10)}{5} = \frac{10 + 2}{3}\)
- \(-8 + \frac{30}{4} = 12\)
- \(-\frac{2}{5} + 6 = 4\) It checks!

Why is Checking so very Important?

- b/c of Extraneous Solutions (false solutions)

Examples

\[a) \frac{1}{3} + \frac{1}{3x} = \frac{1}{6}\]

\[b) \frac{4}{c} = \frac{3}{2c} - \frac{1}{5}\]

Examples

\[c) \frac{5}{m} = \frac{2}{m^2} + 2\]

\[d) t - 2 = \frac{8 - 2t}{t - 1}\]

What is a proportion?

Examples
Examples

e) \( \frac{3}{a} = \frac{5}{a-2} \)

Examples

f) \( \frac{n}{5} = \frac{4}{n+1} \)

Examples

\( g) \ \frac{2}{e^2} = \frac{2}{e^2+1} \)

Examples

\( h) \ \frac{1}{t+3} + \frac{4}{t+5} = \frac{2}{t^2+8t+15} \)

Summary

- Multiply every term by LCD. Note: This only works with equations, not expressions.
- Simplify.
- Solve.
- Check.

Applications of Rationals

- Example a) In work with electric circuits, the given formula occurs. Find b, if a = 8 and c = 12.

\[ \frac{1}{a} = \frac{1}{b} + \frac{1}{c} \]
Example

- B) Solve the given physics formula for the specified variable. (*Hint: Think of the other variables as numbers.*)

\[ F = \frac{GMm}{d^2} \] for \( M \)

Example

- c) Solve the chemistry formula for the given variable.

\[ \frac{PV}{T} = \frac{pv}{t} \] for \( T \).