Variables, Expressions, & Equations

Objectives: To evaluate expressions, translate words to algebra, identify solutions of equations, and distinguish between expressions and equations.

Definitions

- **Variable** – a symbol, usually a letter, that represents one or more numbers
- **Algebraic expression** – a math phrase that can include numbers, variables, and operation symbols
- **Equation** – a math sentence with an equal sign

Examples

Find the value of each expression if \( x = 6 \).

\[ a) \ 5x^2 \quad b) \ \frac{4x - 1}{3x} \]

\[ a) = 5(6)^2 \quad b) = \frac{4(6) - 1}{3(6)} \]

\[ = 5(36) \quad = \frac{24 - 1}{18} \]

\[ = 180 \quad = \frac{23}{18} \]

Find the value of each if \( x = 1 \) and \( y = 5 \).

\[ c) \ 2(2x + y) \quad d) \ \frac{x}{5} + \frac{y}{4} \]

\[ c) = 2(2(1) + 5) \quad d) = \frac{1}{5} + \frac{5}{4} \]

\[ = 2(2 + 5) \quad = \frac{4}{20} + \frac{25}{20} \]

\[ = 2(7) \quad = \frac{29}{20} \]

\[ = 14 \quad \text{or} \ 1 \frac{9}{20} \]

Definitions

- **Addition** – sum, more than, increase
- **Subtraction** – difference, minus, less than, decrease
- **Multiplication** – product, times, of
- **Division** – quotient, per

Replace “a number” with a variable.
Examples

- Change the words to an algebra statement.
  e) 13 added to a number
  f) 8 subtracted from a number
  g) 2 times a number plus 9
  h) 7 less than three times a number

Answers

- e) $13 + x$
- f) $x - 8$
- g) $2x + 9$
- h) $3x - 7$

Definition

- **Equation** - math sentence with an equal sign
- **Solution** - a number that makes an equation true

Note: To determine if a number is a solution, substitute the number for the variable and “do the math”.

Example

- Decide whether the given number is a solution of the equation.
  i) Is 10 a solution of $x + 6 = 15$?
  Does $10 + 6 = 15$? No, so 10 is not a solution.

Another Example

- j) Is $\frac{1}{5}$ a solution of $6x + 4x + 9 = 11$?

  $\text{SOLUTION}$
  - $6 \cdot \frac{1}{5} + 4 \cdot \frac{1}{5} + 9 = 11$?
  - $\frac{6}{5} + 9 = 11$
  - $10 + 9 = 11$
  - $\frac{10}{5} + 9 = 11$
  - $2 + 9 = 11$
  - This is true so $\frac{1}{5}$ is a solution of the equation.

Translating Words to Algebra

- Remember:
  The word is means equal to.

EXAMPLES

- Change each sentence to an equation. Use $x$ to represent the number.
  k) A number minus three equals 1.
     Answer: $x - 3 = 1$
  l) Twelve divided by a number equals $\frac{1}{3}$ times that number.
     Answer: $\frac{12}{x} = \frac{1}{3}x$
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<td>m)</td>
<td>5y - (3y + 6)</td>
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<td>m) expression</td>
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<td>n)</td>
<td>9r + 3(r - 4) = 2</td>
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