Simplifying Radicals

Objective: To simplify radical expressions using the product and quotient rules.

Product Rule for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and n is a natural number, then

$$\sqrt[n]{a}\left(\sqrt[n]{b}\right)=\sqrt[n]{ab}.$$

In other words, the product of two radicals is the radical of the product.

Product Rule Practice

$a)\ \sqrt{10}\left(\sqrt{3}\right)$

$b)\ \sqrt{23}\left(\sqrt{t}\right)$

$c)\ \sqrt[3]{7x}\left(\sqrt[3]{2y}\right)$

$d)\ \sqrt[4]{3y^2}\left(\sqrt[4]{6yz}\right)$

$e)\ \sqrt[6]{5a}\left(\sqrt[5]{4ab}\right)$

SOLUTIONS

$a)\ \sqrt{10}\left(\sqrt{3}\right)=\sqrt{30}$

$b)\ \sqrt{23}\left(\sqrt{t}\right)=\sqrt{23t}$

$c)\ \sqrt[3]{7x}\left(\sqrt[3]{2y}\right)=\sqrt[3]{14xy}$

$d)\ \sqrt[4]{3y^2}\left(\sqrt[4]{6yz}\right)=\sqrt[4]{18y^3z}$

$e)\ \sqrt[6]{5a}\left(\sqrt[5]{4ab}\right)=\text{cannot multiply}$

Quotient Rule for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$, and n is a natural number, then

$$\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}.$$

In other words, the quotient of two radicals is the radical of the quotient.

Quotient Rule Practice

$f)\ \sqrt{\frac{16}{49}}$

$g)\ \sqrt[3]{\frac{216}{27}}$

$h)\ \sqrt[3]{-\frac{8}{125}}$

$i)\ -\sqrt[4]{\frac{625}{16}}$

SOLUTIONS

$f)\ \sqrt{\frac{16}{49}} = \frac{4}{7}$

$g)\ \sqrt[3]{\frac{216}{27}} = \frac{6}{3} = 2$

$h)\ \sqrt[3]{-\frac{8}{125}} = -\frac{2}{5}$

$i)\ -\sqrt[4]{\frac{625}{16}} = -\frac{5}{2}$

Special Note

- **For both the product and quotient properties, you must have the <u>same</u> index!**

Rules for Simplifying Radicals

1. No factor under the radical has a higher power than the root.
2. No <u>fractions</u> under radical.
3. No radical in <u>denominator</u>.
4. No <u>common</u> <u>factors</u> between root and exponents in radical.

Removing Perfect-Square Factors

- Re-write the radicand as a <u>product</u> of **perfect-square factors** times any remaining factors.

For example: $\sqrt{75} = \sqrt{25}\left(\sqrt{3}\right) = 5\sqrt{3}$

Practice Simplifying Radicals

$i)\sqrt{48}$

$j)\ -\sqrt{24}$

$k)\ -5\sqrt{300}$

$l)\ \sqrt{18}$

$m)\ \sqrt[3]{24}$

SOLUTIONS

$i)\sqrt{48} = \sqrt{16 \bullet 3} = 4\sqrt{3}$

$j)\ -\sqrt{24} = -\sqrt{4 \bullet 6} = -2\sqrt{6}$

$k)\ -5\sqrt{300} = -5\sqrt{100 \bullet 3} = -5(10)\sqrt{3} = -50\sqrt{3}$

$l)\ \sqrt{18} = \sqrt{9 \bullet 2} = 3\sqrt{2}$

$m)\ \sqrt[3]{24} = \sqrt[3]{8 \bullet 3} = 2\sqrt[3]{3}$

Practice Simplifying Radicals

$n)\ \sqrt{-150}$

$o)\ \sqrt[3]{-250}$

$p)\ -\sqrt[4]{1250}$

$q)\ \sqrt[5]{128}$

SOLUTIONS

$n)\ \sqrt{-150} = \text{not real}$

$o)\ \sqrt[3]{-250} = \sqrt[3]{-125 \bullet 2} = -5\sqrt[3]{2}$

$p)\ -\sqrt[4]{1250} = -\sqrt[4]{625 \bullet 2} = -5\sqrt[4]{2}$

$q)\ \sqrt[5]{128} = \sqrt[5]{32 \bullet 4} = 2\sqrt[5]{4}$

Removing Variable Factors

- Divide variable exponents by the root and leave any remainder under the radical.

For example: $\sqrt{x^{11}} = x^{\frac{11}{2}} = x^5 x^{\frac{1}{2}} = x^5\sqrt{x}$

Practice with Variables

$r)\sqrt{18m^2}$

$s)\ \sqrt{256z^{12}}$

$t)\ -\sqrt{200p^{13}}$

SOLUTIONS

$r)\sqrt{18m^2} = \sqrt{9 \bullet 2 \bullet m^2} = 3m\sqrt{2}$

$s)\ \sqrt{256z^{12}} = 16z^6$

$t)\ -\sqrt{200p^{13}} = -\sqrt{100 \bullet 2 \bullet p^{12} \bullet p} = -10p^6\sqrt{2p}$

Practice with Variables

$u)\ -\sqrt[3]{216y^{15}x^6z^3}$

$v)\ \sqrt{23k^9p^{14}}$

$w)\ -\sqrt[4]{32k^5m^{10}}$

SOLUTIONS

$u)\ -\sqrt[3]{216y^{15}x^{6}z^{3}} = -6y^{5}x^{2}z$

$v)\ \sqrt{23k^{9}p^{14}} = \sqrt{23k^{8}\bullet k\bullet p^{14}} = k^{4}p^{7}\sqrt{23k}$

$w)\ -\sqrt[4]{32k^{5}m^{10}} = -\sqrt[4]{16\bullet 2\bullet k^{4}\bullet k\bullet m^{8}\bullet m^{2}} = -2km^{2}\sqrt[4]{2m^{2}k}$