Simplifying Radicals

Objective: To simplify radical expressions using the product and quotient rules.

Product Rule for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $n$ is a natural number, then

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}.$$ 

In other words, the product of two radicals is the radical of the product.

SOLUTIONS

a) $\sqrt{10} \cdot \sqrt{3} = \sqrt{30}$
b) $\sqrt{23} \cdot \sqrt{1} = \sqrt{23}$
c) $\sqrt{7x} \cdot \sqrt{2y} = \sqrt{14xy}$
d) $\sqrt{3y^2} \cdot \sqrt{6yz} = \sqrt{18y^3z}$
e) $\sqrt[4]{a} \cdot \sqrt{4ab} = \text{cannot multiply}$

Quotient Rule for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$, and $n$ is a natural number, then

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}.$$ 

In other words, the quotient of two radicals is the radical of the quotient.

Quotient Rule Practice

f) $\sqrt[4]{\frac{16}{49}}$
g) $\sqrt[3]{\frac{216}{27}}$
h) $\sqrt[5]{\frac{8}{125}}$
i) $\sqrt[6]{\frac{625}{16}}$
SOLUTIONS

\[ f) \sqrt[3]{16} = \frac{4}{7} \]
\[ g) \sqrt{216} = \frac{6}{3} = 2 \]
\[ h) \sqrt[3]{8} = \frac{2}{5} \]
\[ i) -\sqrt[3]{625} = -\frac{5}{2} \]

Special Note

- For both the product and quotient properties, you must have the same index!

Rules for Simplifying Radicals

1. No factor under the radical has a higher power than the root.
2. No fractions under radical.
3. No radical in denominator.
4. No common factors between root and exponents in radical.

Removing Perfect-Square Factors

- Re-write the radicand as a product of perfect-square factors times any remaining factors.

For example: \( \sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3} \)

Practice Simplifying Radicals

\[ i) \sqrt{48} \]
\[ j) -\sqrt{24} \]
\[ k) -5\sqrt{300} \]
\[ l) \sqrt{18} \]
\[ m) \frac{3}{2}\sqrt{24} \]

SOLUTIONS

\[ i) \sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3} \]
\[ j) -\sqrt{24} = -\sqrt{4 \cdot 6} = -2\sqrt{6} \]
\[ k) -5\sqrt{300} = -5\sqrt{100 \cdot 3} = -5(10)\sqrt{3} = -50\sqrt{3} \]
\[ l) \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2} \]
\[ m) \frac{3}{2}\sqrt{24} = \frac{3}{2}\sqrt{8 \cdot 3} = 2\sqrt{3} \]
Practice Simplifying Radicals

n) \( \sqrt{-150} \) is not real

o) \( \sqrt{-250} = \sqrt{-125 \cdot 2} = -5\sqrt{2} \)

p) \( -\sqrt{1250} = -\sqrt{625 \cdot 2} = -5\sqrt{2} \)

q) \( \sqrt{128} = \sqrt{32 \cdot 4} = 2\sqrt{4} \)

Removing Variable Factors

- Divide variable exponents by the root and leave any remainder under the radical.

For example: \( \sqrt{x^{11}} = x^{\frac{11}{2}} = x^5 \cdot x^{\frac{1}{2}} = x^5 \sqrt{x} \)

Practice with Variables

r) \( \sqrt{18m^2} \)

s) \( \sqrt{256z^{12}} \)

\( r - \sqrt{200p^{13}} = -\sqrt{100 \cdot 2p^{12}} \cdot p = -10p^6 \sqrt{2p} \)

SOLUTIONS

Practice with Variables

u) \( -\sqrt{216y^{14}x^2z^3} \)

v) \( \sqrt{23k^7p^{15}} \)

w) \( -\sqrt{32k^8m^{16}} \)
SOLUTIONS

u) \(-\sqrt{216} y^6 x^4 z^3 = -6 y^3 x^2 z\)

v) \(\sqrt{23k^9 p^{16}} = \sqrt{23k^8 \cdot p^{16}} = k^4 p^8 \sqrt{23k}\)

w) \(-\sqrt{32k^6 m^{12}} = -\sqrt{16 \cdot 2k^6 \cdot m^{12}} = -2km^2 \sqrt{2m^2 k}\)