Equations of Lines

Today’s objective: To write and graph lines in slope-intercept form and point-slope form.

Vocabulary

- **Linear equation**: an equation whose graph is a line
- **Y-intercept**: the y-coordinate of the point where a line crosses the y-axis
- **Slope-intercept form**: $y = mx + b$
  where $m =$ slope and $b =$ y-intercept

Find the slope and y-intercept of each equation.

a) $y = -2x + 1$
   - $m = -2$
   - $y$-int = 1

b) $y = -(4/5)x$
   - $m = -4/5$
   - $y$-int = 0

c) $y = (7/6)x - 3/4$
   - $m = 7/6$
   - $y$-int = -3/4

Example

d) Write an equation of a line with slope -3 and y-intercept (0, 2).

Example -- SOLUTION

d) Write an equation of a line with slope -3 and y-intercept (0, 2).

Substitute into $y = mx + b$ form.

$y = -3x + 2$
Graphing Lines using the Slope and y-intercept

1. Plot the y-intercept.
2. From this point, use the slope and rise/run to plot a second point.
3. Repeat step 2 to get a third point.
4. Draw a line through the three points.

Example

Solution

Write an equation in slope-intercept form through (7, -2) with slope = \( \frac{1}{4} \).

Begin with point-slope form: \( y - y_1 = m(x - x_1) \)

Substitute slope and point from problem.

\[
\begin{align*}
y - (-2) &= \frac{1}{4}(x - 7) \\
y + 2 &= \frac{1}{4}x - \frac{7}{4} \\
y &= \frac{1}{4}x - \frac{7}{4} - 2 \\
y &= \frac{1}{4}x - \frac{15}{4}
\end{align*}
\]

POINT-SLOPE FORM

\[
y - y_1 = m(x - x_1)
\]

Remember: \( m = \text{slope} \) & \( (x_1, y_1) = \text{a point on the line} \).

Example

f) Write an equation in standard form through (7, -2) with slope = \( \frac{1}{4} \).
Example
Solution

q) Find an equation of the line through the points (-2,2) and (4,3). Write in slope-intercept form. First find the slope using the slope formula.

\[ \frac{3-2}{4-(-2)} = \frac{1}{6} \]

Now, use point-slope form: \( y - y_1 = m(x - x_1) \)
Substitute slope and either point from problem.

\[ y - 3 = \frac{1}{6}(x - 4) \]

(continued)

Example
Solution Continued

\[ y - 3 = \frac{1}{6}(x - 4) \]
\[ y - 3 = \frac{1}{6} x - \frac{4}{6} \]
\[ y = \frac{1}{6} x - \frac{2}{6} + 3 \]
\[ y = \frac{1}{6} x + \frac{7}{3} \]

Steps for Writing an Equation

1) Find the slope.
Remember the 4 special cases:
vertical, horizontal, parallel, perpendicular

2) Using the given information, write an equation in the requested form.
Slope-intercept \( y = mx + b \)
Point-slope \( y - y_1 = m(x - x_1) \)

Write an equation.

h) thru (-2,8) and \( m = 0 \)

i) thru (1, \( \frac{3}{4} \)) and vertical

Write an equation.

SOLUTION

h) thru (-2,8) and \( m = 0 \)
If the slope is zero, then it is a horizontal line with equation \( y = b \). The \( y \) value of the given point is 8 so \( y = 8 \) is the equation.

i) thru (1, \( \frac{3}{4} \)) and vertical
Since it is a vertical line with equation \( x = a \), the \( x \) value of the given point is 1 so \( x = 1 \) is the equation.