4.5 Graphs of Sine and Cosine Functions

Objective 1: Understand the graph of $y = \sin x$. 

Table 4.3 Values of $(x, y)$ on the graph of $y = \sin x$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$0$</th>
<th>$\frac{\pi}{6}$</th>
<th>$\frac{\pi}{3}$</th>
<th>$\frac{\pi}{4}$</th>
<th>$\frac{\pi}{2}$</th>
<th>$\frac{5\pi}{6}$</th>
<th>$\pi$</th>
<th>$\frac{7\pi}{6}$</th>
<th>$\frac{5\pi}{4}$</th>
<th>$\frac{3\pi}{2}$</th>
<th>$\frac{4\pi}{3}$</th>
<th>$\frac{2\pi}{3}$</th>
<th>$\frac{\pi}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \sin x$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\sqrt{3}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>1</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$-\frac{\sqrt{3}}{2}$</td>
<td>$-1$</td>
<td>$-\frac{\sqrt{2}}{2}$</td>
<td>$-\frac{1}{2}$</td>
<td>0</td>
</tr>
</tbody>
</table>

- As $x$ increases from $0$ to $\frac{\pi}{2}$, $y$ decreases from $0$ to $1$.
- As $x$ increases from $\frac{\pi}{2}$ to $\pi$, $y$ decreases from $1$ to $0$.
- As $x$ increases from $\pi$ to $\frac{3\pi}{2}$, $y$ increases from $0$ to $-1$.
- As $x$ increases from $\frac{3\pi}{2}$ to $2\pi$, $y$ decreases from $-1$ to $0$.

$y = \sin x, \ 0 \leq x < 2\pi$

Period: $2\pi$
Continuation of the previous problem showing 3 cycles

Note: the pattern repeats in every interval of length $2\pi$.

Therefore the period for $y = \sin x$ is $2\pi$.

Domain: $(-\infty, \infty)$  Range: $[-1, 1]$  

This is an odd function – it is symmetric to the origin.

Objective 2: Graph variations of $y = \sin x$.

$Y = A \sin x$

Using the graph of $y = \sin x$, multiply each $y$-value by $A$.

Graphing $y=2 \sin x$

$|A|$ is called the amplitude of $y = A \sin x$.  

Figure 4.65: Comparing the graphs of $y = \sin x$ and $y = 2 \sin x$. 
Determine the amplitude of each function. Then graph the function and $y = \sin x$ in the same rectangular coordinate system for $0 \leq x \leq 2\pi$. 

$Y = 5 \sin x$

$Y = \frac{1}{4} \sin x$
$y = -4 \sin x$

**Amplitudes and Periods**

The graph of $y = A \sin Bx$ has

- **amplitude** = $|A|$
- **period** = $\frac{2\pi}{B}$.
Determine the amplitude and period of each function. Then graph one period of the function.

\[ Y = \sin 4x \]

\[ y = 2 \sin \frac{1}{4}x \]
$Y = 3 \sin 2\pi x$

$y = A \sin (Bx - C) \quad Y = \sin (x - \pi/2)$

Horizontal shift
\[ Y = \sin (2x - \pi/2) \]

\[ Y = -3 \sin (2x + \pi/2) \]
Objective 3: Understand the graph of \( y = \cos x \).

Table 4.4 Values of \((x, y)\) on the graph of \( y = \cos x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>( \frac{\pi}{6} )</th>
<th>( \frac{\pi}{3} )</th>
<th>( \frac{\pi}{2} )</th>
<th>( \frac{5\pi}{6} )</th>
<th>( \pi )</th>
<th>( \frac{7\pi}{6} )</th>
<th>( \frac{4\pi}{3} )</th>
<th>( \frac{5\pi}{3} )</th>
<th>( \frac{3\pi}{2} )</th>
<th>( 2\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \cos x )</td>
<td>1</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>-1</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

- As \( x \) increases from 0 to \( \frac{\pi}{2} \), \( y \) decreases from 1 to 0.
- As \( x \) increases from \( \frac{\pi}{2} \) to \( \pi \), \( y \) increases from 0 to -1.
- As \( x \) increases from \( \pi \) to \( \frac{3\pi}{2} \), \( y \) decreases from -1 to 0.
- As \( x \) increases from \( \frac{3\pi}{2} \) to \( 2\pi \), \( y \) increases from 0 to 1.

The range is \(-1 \leq y \leq 1\).

Period: \( 2\pi \)

Y = \cos x

Amplitude = 1, Period = \( 2\pi \)

Domain: \((-\infty, \infty)\), Range: \([-1, 1]\)

Even function – symmetric to the y-axis

Objective 4:

Graph variations of \( y = \cos x \).

\[ Y = 3 \cos x \]
Determine the amplitude, period, and phase shift of $y=2\cos(3x - \pi)$.
Vertical shifts

\[ y = A \sin (Bx - C) + D \]

D is a constant which causes a vertical shift of D units. If D is positive, it is an upward shift. If D is negative, it is a downward shift.

\[ y = \sin x - 2 \]

\[ Y = 2 \cos \frac{1}{2}x + 1 \]
Find an equation for each graph

Amp = 3
Since it goes to pos 3,
A = 3

Period = 4π and since period = 2π/B,
B = 2π/period
B = 2π/4π, B = 1/2

Y = 3 sin ½ x