Polynomial and Rational Inequalities

Objectives
- Solve polynomial inequalities.
- Solve rational inequalities.

Procedure for Solving Polynomial Inequalities

1. Express the inequality in the form \( f(x) > 0 \) or \( f(x) < 0 \) where \( f \) is a polynomial function.
2. Solve the equation \( f(x) = 0 \). The real solutions are the boundary points.
3. Locate these boundary points on a number line, dividing the number line into intervals.

Procedure for Solving Polynomial Inequalities (cont.)

4. Choose one representative number, called a test value, within each interval and evaluate \( f \) at that number.
   a. If the value of \( f \) is positive, then \( f(x) > 0 \) for all numbers, \( x \), in the interval.
   b. If the value of \( f \) is negative, then \( f(x) < 0 \) for all numbers, \( x \), in the interval.
5. Write the solution set, selecting the interval or intervals that satisfy the given inequality.

This procedure is valid if \(<\) is replaced by \(\leq\) or \(>\) replaced by \(\geq\). However, if the inequality involves \(\leq\) or \(\geq\) include the boundary points in the solution set.
Example
• Solve $3x^2 + 16x < -5$

Solving Rational Inequalities
• VERY similar to solving polynomial inequalities EXCEPT if the denominator equals zero, there is a domain restriction. The function COULD change signs on either side of that point.

• Step 1: Compare inequality to zero. (Add constant to both sides and use a common denominator to have a rational expression.)

• Step 2: Factor both numerator & denominator to find “cut-off” values for regions to check when function becomes positive or negative.

Example
• Solve $\frac{x + 3}{x - 1} \geq 0$