Polynomial Functions and Their Graphs

End Behavior

Smooth, Continuous Graphs

Definition of a Polynomial Function

Let \( n \) be a nonnegative integer and let \( a_0, a_1, \ldots, a_{n-1}, a_n \) be real numbers, with \( a_n \neq 0 \). The function defined by

\[
f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0
\]

is called a polynomial function of degree \( n \). The number \( a_n \), the coefficient of the variable to the highest power, is called the leading coefficient.

Polynomial functions of degree 2 or higher have graphs that are smooth and continuous. By **smooth**, we mean that the graphs contain only rounded curves with no sharp corners. By **continuous**, we mean that the graphs have no breaks and can be drawn without lifting your pencil from the rectangular coordinate system.

Graphs of Polynomial Functions and Nonpolynomial Functions

End Behavior of Polynomial Functions
Example

Use the Leading Coefficient Test to determine the end behavior of the graph of

\( f(x) = -3x^2 - 4x + 7 \)

Use the Leading Coefficient Test to determine the end behavior of the graph of

\( f(x) = -2x^4(x-1)^3(x+3) \)

Determine the end behavior of

\( f(x) = (x+1)^2(x^2-9) \)

Which function could possibly coincide with this graph?

1) \(-7x^5 + 5x + 1\)
2) \(9x^5 + 5x^2 - 7x + 1\)
3) \(3x^4 + 2x^2 + 1\)
4) \(-4x^4 + 2x^2 + 1\)