Logarithmic Functions

Graphing

Objectives

– Graph logarithmic functions.
– Find the domain of a logarithmic function.

Flashback

Remember the graph of the exponential function $y = f(x) = 3^x$.

Logarithmic Functions

• Logarithmic functions are inverses of exponential functions.

To graph a logarithmic equation, convert to exponential form.
1. Choose values for $y$.
2. Compute values for $x$.
3. Plot the points and connect them with a smooth curve.

* Note that the curve does not touch or cross the $y$-axis.

Example

• Graph $y = \log_3 x$

<table>
<thead>
<tr>
<th>$x = 3^y$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>(1, 0)</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>(3, 1)</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>(9, 2)</td>
</tr>
<tr>
<td>1/3</td>
<td>−1</td>
<td>(1/3, −1)</td>
</tr>
<tr>
<td>1/9</td>
<td>−2</td>
<td>(1/9, −2)</td>
</tr>
<tr>
<td>1/27</td>
<td>−3</td>
<td>(1/27, −3)</td>
</tr>
</tbody>
</table>
**Side-by-Side Comparison**

\[ f(x) = 3x \quad f(x) = \log_3 x \]

**Comparing Exponential and Logarithmic Functions**

- **Logarithmic Functions**
  - Logarithmic functions are inverses of exponential functions.
  - The inverse of \( f(x) = a^x \) is given by \( f^{-1}(x) = \log_a x \)

- **Asymptotes**
  - Recall that the horizontal asymptote of the exponential function \( y = a^x \) is the x-axis.
  - The horizontal axis of a logarithmic function \( y = \log_a x \) is the y-axis.
Graphs of Logarithmic Functions

- Graph: \( y = f(x) = \log_b x \).
  - Select \( y \).
  - Compute \( x \).

<table>
<thead>
<tr>
<th>( x, \text{ or } 6^y )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>36</td>
<td>2</td>
</tr>
<tr>
<td>216</td>
<td>3</td>
</tr>
<tr>
<td>1/6</td>
<td>−1</td>
</tr>
<tr>
<td>1/36</td>
<td>−2</td>
</tr>
</tbody>
</table>

Transformations of logarithmic functions are treated as other transformations.

- Follow order of operations.
- Note: When graphing a logarithmic function, the graph only exists for \( x > 0 \).
- WHY?
  - If a positive number is raised to an exponent, no matter how large or small, the result will always be POSITIVE!

Example

- Graph each of the following.
- Describe how each graph can be obtained from the graph of \( y = \ln x \).
- Give the domain and the vertical asymptote of each function.

  a) \( f(x) = \ln (x - 2) \)
  - The graph is a shift 2 units right.
  - The domain is the set of all real numbers greater than 2.
  - The line \( x = 2 \) is the vertical asymptote.

  b) \( f(x) = |\ln (x + 1)| \)
  - The graph is a translation 1 unit to the left.
  - Then the absolute value has the effect of reflecting negative outputs across the \( x \)-axis.
  - The domain is the set of all real numbers greater than \(-1 \).
  - The line \( x = -1 \) is the vertical asymptote.

Application: Walking Speed

- In a study by psychologists Bornstein and Bornstein, it was found that the average walking speed \( w \), in feet per second, of a person living in a city of population \( P \), in thousands, is given by the function \( w(P) = 0.37 \ln P + 0.05 \).
- If the population of Salem, Oregon is 137,000, what is the average walking speed of a person living in that city?