Basics of Functions –
What is a Function?

Relations

Example

Find the domain and the range.

\{(98.6, Felicia), (98.3, Gabriella), (99.1, Lakesha)\}

Definition of a Relation

A relation is any set of ordered pairs. The set of all first components of the ordered pairs is called the domain of the relation and the set of all second components is called the range of the relation.

\{(sitting, 80), (walking, 325), (aerobics, 505), (tennis, 505), (running, 720), (swimming, 790)\}

Domain: \{sitting, walking, aerobics, tennis, running, swimming\}

Range: \{80, 325, 505, 720, 790\}

Do not list 505 twice.

Four Ways to Represent a Function

- Verbally (description in words)
- Numerically (by a table of values)
- Visually (by a graph)
- Algebraically (by a formula)
**A Function Verbally**

ex. The amount of sales tax depends on the amount of the purchase.

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**A Function Numerically**

Use a table of values:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>19</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>-1</td>
<td>7</td>
</tr>
</tbody>
</table>

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**A Function Visually**

Any function can be visually represented by a graph.

![Graph](image.png)

**A Function Algebraically**

\[ f(x) = 3x + 10 \]

To solve a function: Evaluate the function.

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A relation in which each member of the domain corresponds to exactly one member of the range is a function. Notice that more than one element in the domain can correspond to the same element in the range. Aerobics and tennis both burn 505 calories per hour.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Calories Burned per hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sitting</td>
<td>80</td>
</tr>
<tr>
<td>Walking</td>
<td>325</td>
</tr>
<tr>
<td>Aerobics</td>
<td>505</td>
</tr>
<tr>
<td>Tennis</td>
<td>720</td>
</tr>
<tr>
<td>Running</td>
<td>790</td>
</tr>
</tbody>
</table>

Is this a function? Does each member of the domain correspond to precisely one member of the range? This relation is not a function because there is a member of the domain that corresponds to two members of the range. 505 corresponds to aerobics and tennis.
Determine whether each relation is a function.

\{(1, 8), (2, 9), (3, 10)\}

\{(2, 3), (2, 4), (2, 5)\}

\{(3, 6), (4, 6), (5, 6)\}

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Here is an equation that models paid vacation days each year as a function of years working for the company.

\[ y = 0.016x^2 + 0.93x + 8.5 \]

The variable \( x \) represents years working for a company. The variable \( y \) represents the average number of vacation days each year. The variable \( y \) is a function of the variable \( x \). For each value of \( x \), there is one and only one value of \( y \). The variable \( x \) is called the independent variable because it can be assigned any value from the domain. Thus, \( x \) can be assigned any positive integer representing the number of years working for a company. The variable \( y \) is called the dependent variable because its value depends on \( x \). Paid vacation days depend on years working for a company.

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Not every set of ordered pairs defines a function. Not all equations with the variables \( x \) and \( y \) define a function. If an equation is solved for \( y \) and more than one value of \( y \) can be obtained for a given \( x \), then the equation does not define \( y \) as a function of \( x \). So the equation is not a function.

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Determine whether each equation defines \( y \) as a function of \( x \).

\[ x + 4y = 8 \]

\[ x^2 + 2y = 10 \]

\[ x^2 + y^2 = 16 \]
Function Notation

The special notation \( f(x) \), read "f of x" or "f at x" represents the value of the function at the number \( x \).

If a function named \( f \) and \( x \) represents the independent variable, the notation \( f(x) \) corresponds to the y-value for a given \( x \).

\[ f(x) = -0.016x^2 + 93x + 8.5 \]

This is read "f of x equals -0.016x^2 + 93x + 8.5"

Example

Evaluate each of the following.

Find \( f(3) \) for \( f(x) = 2x^2 - 4 \)

Find \( f(-2) \) for \( f(x) = -9 - x^2 \)

Graphs of Functions

We are evaluating the function at 0 when we substitute 0 for \( x \) as we see below.

\[ f(0) = -0.016(0)^2 + 93(0) + 8.5 \]

What is the answer?

The graph of a function is the graph of its ordered pairs. First find the ordered pairs, then graph the functions.

Graph the functions \( f(x) = 2x \) and \( g(x) = -2x + 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = -2x )</th>
<th>( (x, y) )</th>
<th>( g(x) = -2x + 3 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( f(-2) = 4 )</td>
<td>(-2, 4)</td>
<td>( g(-2) = 7 )</td>
<td>(-2, 7)</td>
</tr>
<tr>
<td>-1</td>
<td>( f(-1) = 2 )</td>
<td>(-1, 2)</td>
<td>( g(-1) = 5 )</td>
<td>(-1, 5)</td>
</tr>
<tr>
<td>0</td>
<td>( f(0) = 0 )</td>
<td>(0, 0)</td>
<td>( g(0) = 3 )</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>1</td>
<td>( f(1) = -2 )</td>
<td>(1, -2)</td>
<td>( g(1) = 1 )</td>
<td>(1, 1)</td>
</tr>
<tr>
<td>2</td>
<td>( f(2) = -4 )</td>
<td>(2, -4)</td>
<td>( g(2) = -1 )</td>
<td>(2, -1)</td>
</tr>
</tbody>
</table>

See the next slide.
Graph the following functions: $f(x) = 3x - 1$ and $g(x) = 3x$.

The Vertical Line Test

Example

Use the vertical line test to identify graphs in which $y$ is a function of $x$. 

The first graph is a function, the second is not. 

Example 

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