Matrices

The Inverse of a Matrix

What is a Matrix Inverse?

• The inverse of a matrix is comparable to the reciprocal of a real number.

• The product of a matrix and its identity matrix is always the matrix itself. In other words, multiplying a matrix by its identity matrix is like multiplying a number by 1.

Determining if Matrices are Inverses of Each Other

• Recall that a number multiplied by its multiplicative inverse yields a product of 1.

• Similarly, the product of matrix $A$ and its multiplicative inverse matrix $A^{-1}$ (read “$A$-inverse”) is $I$, the identity matrix.

• So, to prove that two matrices are inverses of each other, show that their product, regardless of the order they’re multiplied, is always the identity matrix.

Finding the Inverse of a 2 x 2 Matrix

If a matrix is of the form

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

then the inverse can be found by calculating:

$$\frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Note: $ad - bc \neq 0$.

Example

• Find the inverse, if it exists, for the matrix.

$$\begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix}$$
Solving a Matrix Equation

If the matrix $A$ has an inverse, then the solution of the matrix equation $AX = B$ is given by $X = A^{-1}B$.

Example: Solving a Matrix Equation

• Use a matrix equation to solve

\[
\begin{align*}
-2x + 4y &= 2 \\
-3x + 7y &= 7
\end{align*}
\]