Linear Equations & Straight Lines

The Slope of a Straight Line

Example: Slope of $y = mx + b$
Find the slope.
- $y = 6x - 9$
- $y = -x + 4$
- $y = 2$
- $x = 3$

Geometric Definition of Slope

Let $L$ be a line passing through the points $(x_1, y_1)$ and $(x_2, y_2)$ where $x_1 \neq x_2$. Then the slope of $L$ is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$ 

Example
Find the slope of the line passing through (-2, -3) and (-1,6).

Point-Slope Formula

The equation of the straight line through the point $(x_1, y_1)$ and having slope $m$ is given by

$$y - y_1 = m(x - x_1).$$
Example: Point-Slope Formula
Find the equation of the line through the point (-1,4) with a slope of $\frac{3}{5}$.

Perpendicular Property

- **Perpendicular Property** When two lines are perpendicular, their slopes are negative reciprocals of one another. That is, if two lines with slopes $m$ and $n$ are perpendicular to one another, then
  
  \[ m = -\frac{1}{n}. \]

- Conversely, if two lines have slopes that are negative reciprocals of one another, they are perpendicular.

Example: Perpendicular Property
Find the equation of the line through the point (3,-5) that is perpendicular to the line whose equation is $2x + 4y = 7$.

Parallel Property

- **Parallel Property** Parallel lines have the same slope. Conversely, if two lines have the same slope, they are parallel.

Example: Parallel Property
Find the equation of the line through the point (3,-5) that is parallel to the line whose equation is $2x + 4y = 7$. 
Cost Analysis

- The cost of manufacturing an item commonly consists of two parts: the **fixed cost** and the **cost per item**.
- The fixed cost is constant (for the most part) and doesn’t change as more items are made.
- The total value of the second cost does depend on the number of items made.

Marginal Cost

- In economics, **marginal cost** is the rate of change of cost $C(x)$ at a level of production $x$ and is equal to the slope of the cost function at $x$.
- The marginal cost is considered to be constant with linear functions.

Cost Function

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<tr>
<th>COST FUNCTION</th>
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<td>In a cost function of the form $C(x) = mx + b$, the $m$ represents the marginal cost per item and $b$ the fixed cost. Conversely, if the fixed cost of producing an item is $b$ and the marginal cost is $m$, then the cost function $C(x)$ for producing $x$ items is $C(x) = mx + b$.</td>
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Break-Even Analysis

- A profit can be made only if the revenue received from its customers exceeds the cost of producing and selling its goods and services.
- The number of units $x$ at which revenue just equals cost is the **break-even quantity**; the corresponding ordered pair gives the **break-even point**.

Break-Even Analysis

- The **revenue** $R(x)$ from selling $x$ units of an item is the product of the price per unit $p$ and the number of units sold (demand) $x$, so that $R(x) = px$.
- The corresponding **profit** $P(x)$ is the difference between revenue $R(x)$ and cost $C(x)$.
  $$P(x) = R(x) - C(x)$$

Break-Even Point

- As long as revenue just equals cost, the company, etc. will break even (no profit and no loss).
  $$R(x) = C(x)$$
Example
• Suppose that the total cost of making \( x \) coats is given by the function \( y = 40x + 2400 \).

a.) What is the cost of making 100 coats?

b.) How many coats can be made for $3600?

c.) Find and interpret the \( y \)-intercept of the graph of the equation.

d.) Find and interpret the slope of the graph of the equation.

Example
• The same coat factory in the previous example sells coats for $100 each.

a.) State the revenue function.

b.) What is the revenue if 300 coats are sold?

c.) Using the cost function from the previous example, state the profit function for the coat factory.

d.) How much profit would be made if 80 coats are sold?

e.) How many coats must be sold to have a profit of $6000?