MTH 110 Practice Test (Sections 1.3, 1.4, 2.3, 2.4, 8.1, 8.2)

Solve the problem.
  1) Solve the system of linear equations:
     \[
     \begin{cases}
     y = 5x - 3 \\
     y = -3x - 11
     \end{cases}
     \]

  2) Find the point of intersection of the two lines \(x + 3y = 6\) and \(x - y = 2\). Use the elimination method.

Find the equation for the line described.
  3) The line through \((5,2)\) and \((-2,-12)\)

  4) The line having \(y\)-intercept \((0,5)\) and parallel to \(2x + y = 10\)

  5) The line perpendicular to \(y = -4x + 20\) and passing through \((-4,8)\)
6) Suppose a manufacturer finds that the number of units \( x \) she produces and the cost \( y \) of producing \( x \) units are related by an equation of the form \( y = mx + b \). If it costs $2300 to produce 10 units and $2600 to produce 25 units, what does it cost to produce 75 units?

7) Suppose that the supply and demand equations of a certain commodity are given by the following equations, where \( p \) is the unit price of the commodity in dollars and \( q \) is the quantity.

Supply: \( q = 5p - 15 \)  
Demand: \( q = -2.5p + 30 \)

(a) What is the supply when the price is $8?

(b) What is the demand when the price is $8?

(c) Find the equilibrium price and the corresponding number of units supplied and demanded.

(d) Draw the graphs of the supply and demand equations on the same set of axes. Label each equation.
8) A game company has fixed costs of $40,000 per year. Each game costs $11.50 to produce and sells for $20.00. How many games must the company produce and sell each year in order to make a profit of $100,000?

9) The cost to run a TV factory depends on the number of items produced. The factory’s fixed cost is $15,000 each month in addition to $150 for each TV produced.

(a) Write an equation relating $y$, the factory’s costs, to $x$, the number of TVs produced.

(b) When will the factory’s costs reach $25,000?

State whether the calculation is possible. If possible, perform the calculation. If it's not possible to perform the multiplication, explain why – in detail. Remember to show work!!

10) $\begin{pmatrix} 2 & 1 & 0 \\ 3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 7 & -1 \\ 1 & 0 \\ 5 & 2 \end{pmatrix}$

11) $\begin{pmatrix} 1 & 0 & -2 \\ 2 & 4 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$
Are the following matrices inverses of each other? Show mathematical justification and explain your conclusion.

12) \[ A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \]

Find the inverse of the matrix, if it exists.

13) \[ \begin{bmatrix} 2 & -1 \\ -6 & 8 \end{bmatrix} \]

Solve the problem.

14) Consider the system:
\[
\begin{align*}
2x + 3y &= 4 \\
-2x - y &= 8
\end{align*}
\]

(a) Rewrite it in the form \( AX = B \), where \( A \), \( B \), and \( X \) are appropriate matrices.

b) Find the inverse of \( A \).

(c) Solve the system by computing \( A^{-1}B \).
15) In a certain company it has been determined that in one year 10% of the skilled workers become unskilled and 45% of the unskilled workers become skilled.

(a) Write a stochastic (transition) matrix, labeling the rows and columns with $S$ for skilled and $U$ for unskilled, which describes the transitions.

(b) Assume that at the beginning of a year 75% of the workers are skilled and 25% of the workers are unskilled. What percentage of the workers will be unskilled at the beginning of the next year?

16) A survey in a small mining town indicates that 45% of miners' sons become miners, whereas 55% do not. Also 10% of nonminers' sons become miners. At the present time, 70% of the town's men are miners and 30% are nonminers. Considering this as a Markov process and creating a transition matrix below, give the percentage of the town's men who will be miners in the first generation, ...in the second generation. Label the answer for each generation clearly and answer the question each time.

Transition matrix:
Determine whether the matrix is a regular stochastic matrix. Give a reason for your conclusion.

17) \[
\begin{bmatrix}
0.3 & 1 \\
0.7 & 0
\end{bmatrix}
\]

Solve the problem.

18) A national study of collegiate beer drinkers revealed the following facts:
   (I) Among current beer drinkers, 30% prefer one of the "light" beers and 70% prefer "regular" beer.
   (II) In any given year, 20% of the "light" drinkers switch to "regular," and 25% of the "regular" drinkers switch to "light."

(a) Write the initial distribution matrix. (Label the matrix using L for "light" and R for "regular.")

(b) Write the transition matrix for the study. (Label columns and rows using L for "light" and R for "regular.")

(c) After two generations (years), what is the probability that "regular" beer drinkers will switch to "light" beer?

d) In the long run, what proportion of collegiate beer drinkers will drink light beer?