SECTION 8.1

Matrix Solutions to Linear Systems
Graphing Inequalities

- Ex: $x^2 - y^2 > 16$

- Our solution sets are typically \textit{regions} of the Cartesian plane, not just points.

- When graphing . . .

<table>
<thead>
<tr>
<th>Inequality Sign</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq$</td>
<td>Graph a solid curve</td>
</tr>
<tr>
<td>$\leq$</td>
<td></td>
</tr>
<tr>
<td>$&gt;$</td>
<td>Graph a dashed curve</td>
</tr>
<tr>
<td>$&lt;$</td>
<td></td>
</tr>
</tbody>
</table>
Systems of Inequalities

• In order to solve a system of inequalities . . .
  1. Graph the two inequalities.
  2. Don’t forget to pay attention to “solid or dashed.”
  3. Shade the region where they overlap.
Matrices

- Matrices are a valuable tool when used to solve systems of linear equations.

- We can put linear equations into augmented matrices and perform row operations corresponding to the equation manipulations and combinations we have done in the past.

System of Linear Equations

\[
\begin{align*}
3x + y + 2z &= 31 \\
x + y + 2z &= 19 \\
x + 3y + 2z &= 25
\end{align*}
\]

Augmented Matrix

\[
\begin{pmatrix}
3 & 1 & 2 & 31 \\
1 & 1 & 2 & 19 \\
1 & 3 & 2 & 25
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 2 & -5 & -19 \\
0 & 1 & 3 & 9 \\
0 & 0 & 1 & 4
\end{pmatrix}
\]
Example 1

• Write the augmented matrix for the system of equations.

\[
\begin{align*}
\begin{cases}
    x - y + 10z &= 7 \\
    y - 4z &= -5 \\
    z &= 2
\end{cases}
\end{align*}
\]
Example 2

- Write the augmented matrix for the system of equations.

\[
\begin{align*}
2w - 2x + 4y + z &= 16 \\
-2x + y &= -5 \\
w + x - 9y &= -39 \\
-9w - 8x + 2y &= -39
\end{align*}
\]
Example 3

- Write the system of linear equations represented by the augmented matrix. Use $x$, $y$, and $z$ for the variables.

\[
\begin{bmatrix}
7 & 0 & 4 & -13 \\
0 & 1 & -5 & 11 \\
2 & 7 & 0 & 6
\end{bmatrix}
\]
Matrices (cont.)

• Rows in a matrix can be . . .
  1. multiplied by a non-zero constant (represented $kR_i$),
  2. interchanged (represented $R_i \leftrightarrow R_j$), and
  3. added or subtracted (represented $R_i \pm R_j$).

• Two matrices are *row equivalent* if one can be obtained from the other by a sequence of row operations.

• A matrix with 1s down the diagonal and 0s below the 1s is said to be in *row-echelon form*. 
Think back . . .

\[
\begin{align*}
    x - 2y &= 3 \\
    5x + y &= 4
\end{align*}
\]
Example 4

• Perform the indicated matrix row operation and write the new matrix.

\[
3R_1 - 2R_2
\]

\[
\begin{bmatrix}
2 & -6 & 4 & 10 \\
1 & 5 & -5 & 0 \\
3 & 0 & 4 & 7
\end{bmatrix}
\]
Using Matrices to Solve Linear Systems

- We can use matrices, applying *Gaussian Elimination*, to solve linear systems.

### Solving Linear Systems of Three Equations with Three Variables Using Gaussian Elimination

1. Write the augmented matrix for the system.

2. Use matrix row operations to simplify the matrix to a row-equivalent matrix in row-echelon form, with 1s down the main diagonal from upper left to lower right, and 0s below the 1s in the first and second columns.

   \[
   \begin{pmatrix}
   1 & * & * \\
   * & * & * \\
   * & * & *
   \end{pmatrix}
   \rightarrow
   \begin{pmatrix}
   1 & * & * \\
   0 & * & * \\
   * & * & *
   \end{pmatrix}
   \rightarrow
   \begin{pmatrix}
   1 & * & * \\
   0 & 1 & * \\
   0 & * & *
   \end{pmatrix}
   \rightarrow
   \begin{pmatrix}
   1 & * & * \\
   0 & 1 & * \\
   0 & 0 & *
   \end{pmatrix}
   \rightarrow
   \begin{pmatrix}
   1 & * & * \\
   0 & 1 & * \\
   0 & 0 & 1
   \end{pmatrix}
   \]

   - Get 1 in the upper left-hand corner.
   - Use the 1 in the first column to get 0s below it.
   - Get 1 in the second row, second column position.
   - Use the 1 in the second column to get 0 below it.
   - Get 1 in the third row, third column position.

3. Write the system of linear equations corresponding to the matrix in step 2 and use back-substitution to find the system’s solution.
Example 5

Solve the system using matrices. If there is no solution or if there are infinitely many solutions and a system’s equations are dependent, so state.

\[
\begin{align*}
  x + y - z &= -2 \\
  2x - y + z &= 5 \\
  -x + 2y + 2z &= 1
\end{align*}
\]
Example 6

Solve the system using matrices. If there is no solution or if there are infinitely many solutions and a system’s equations are dependent, so state

\[ x - 3z = -1 \]
\[ x + 5y - z = -4 \]
\[ -3x + 6y + 2z = 11 \]
Gauss-Jordan Elimination

- Continues the process until there are 1s on the main diagonal and 0s below \textit{and} above the main diagonal.

- Such a matrix is said to be in \textit{reduced row-echelon form}.

\[
\begin{bmatrix}
1 & 0 & 0 & | & a \\
0 & 1 & 0 & | & b \\
0 & 0 & 1 & | & c \\
\end{bmatrix}
\]

- Either method can be applied to all of our problems.

- We will just focus on Gaussian Elimination in this course.
Example 7

Solve the system using Gaussian Elimination with back-substitution or Gauss-Jordan Elimination.

\[
\begin{align*}
3x + y - z &= 0 \\
2x + 3y - 5z &= 1 \\
x - 2y + 3z &= -4
\end{align*}
\]
Example 8

The nutritional content per ounce of three foods is presented in the table on the right. If a meal consisting of the three foods allows exactly 2200 calories, 110 grams of protein, and 900 milligrams of vitamin C, how many ounces of each kind of food should be used?

<table>
<thead>
<tr>
<th></th>
<th>Calories</th>
<th>Protein (in grams)</th>
<th>Vitamin C (in milligrams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food A</td>
<td>100</td>
<td>9</td>
<td>50</td>
</tr>
<tr>
<td>Food B</td>
<td>400</td>
<td>8</td>
<td>250</td>
</tr>
<tr>
<td>Food C</td>
<td>300</td>
<td>15</td>
<td>100</td>
</tr>
</tbody>
</table>