Section 3.4

Exponential & Logarithmic Functions
Exponential Equations

• An exponential equation is an equation containing a variable in an exponent.

• Ex’s: $2^{3x-8} = 16$, $4^x = 15$, $40e^{0.6x} = 240$

• So, how do we solve exponential equations?
Summary of Method Used in Ex 1

• In Ex 1, we could rewrite both sides of the equation using the same base.

Solving Exponential Equations by Expressing Each Side as a Power of the Same Base

If \( b^M = b^N \), then \( M = N \).

1. Rewrite the equation in the form \( b^M = b^N \).
2. Set \( M = N \).
3. Solve for the variable.
What do we do here?

\[ 4^x = 15 \]

We cannot rewrite both sides in terms of base 2 or base 4.

\[ 10^x = 120,000. \]

We cannot rewrite both sides in terms of base 10.

- Our previous method won’t work on these.
Summary of Method Used in Ex 2

Using Logarithms to Solve Exponential Equations

1. Isolate the exponential expression.
2. Take the natural logarithm on both sides of the equation for bases other than 10. Take the common logarithm on both sides of the equation for base 10.
3. Simplify using one of the following properties:
   \[ \ln b^x = x \ln b \quad \text{or} \quad \ln e^x = x \quad \text{or} \quad \log 10^x = x. \]
4. Solve for the variable.

- This method is used when we can’t rewrite each side using the same base.
One Important Thing to Remember

- The logarithmic function is not defined for zero or negative numbers.
- So, we need to check our solutions!
What about our logarithms?

- A **logarithmic equation** is an equation containing a variable in a logarithmic expression.

- Ex’s:

  \[
  \log_4(x + 3) = 2 \quad \text{and} \quad \ln(4x + 3) = \ln \left( \frac{1}{x} \right)
  \]

- Note: When solving logarithmic equations, we **always** need to check our solution!
Section 3.5

Exponential Growth and Decay; Modeling Data
The Power of Exponential Growth

A Penny Doubled

Would you take a penny doubled every day for a month or $1,000,000?

\[ A = 0.01(2^{n-1}) \]

**After 30 Days:** $1,000,000 vs. $5,368,709.12
Exponential Growth and Decay Models

The mathematical model for **exponential growth** or **decay** is given by

\[ f(t) = A_0 e^{kt} \quad \text{or} \quad A = A_0 e^{kt}. \]

- **If** \( k > 0 \), **the function models the amount, or size, of a growing entity.** \( A_0 \) is the original amount, or size, of the growing entity at time \( t = 0 \), \( A \) is the amount at time \( t \), and \( k \) is a constant representing the growth rate.

- **If** \( k < 0 \), **the function models the amount, or size, of a decaying entity.** \( A_0 \) is the original amount, or size, of the decaying entity at time \( t = 0 \), \( A \) is the amount at time \( t \), and \( k \) is a constant representing the decay rate.

![Graphs of Exponential Growth and Decay](image)
Example 1

a. Find an exponential growth function that models the data for 1970 through 2007.
Example 1 (cont.)

b. By which year will the U.S. population reach 315 million?
Example 2

The exponential model $A = 859.8e^{0.019t}$ describes the population, $A$, of a country in millions, $t$ years after 2003. Use the model to determine the population of the country in 2003.
Example 3

Complete the table for the population growth model for a certain country.

<table>
<thead>
<tr>
<th>2004 Population (millions)</th>
<th>Projected 2020 Population (millions)</th>
<th>Projected Growth Rate, $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>47.3</td>
<td>64.3</td>
<td></td>
</tr>
</tbody>
</table>
Example 4

If you invest $500 in a CD, and it is compounded quarterly at 2.5% annual interest, how long will it take to double? What if it was at 5%?

Don’t forget: $A = P \left(1 + \frac{r}{n}\right)^{nt}$
Example 5

In Example 4, did the amount that you started with make a difference with regard to how long it takes for the investment to double? Why?

Don’t forget: \[ A = P \left(1 + \frac{r}{n}\right)^{nt}\]
Exponential Decay

- Used to describe situations where an amount is decreasing exponentially.

- Often applied to situations where there is some type of decay or deterioration (e.g. radioactive).

- **Carbon-dating** is used to date fossils and artifacts.

- "Half-life" = amount of time it takes for half a substance to disintegrate
Example 6

Given that the half-life of carbon-14 is 5715, find an exponential decay model for carbon-14.
In 1947, earthenware jars containing what are known as the Dead Sea Scrolls were found by an Arab Bedouin herdsman. Analysis indicated that the scroll wrappings contained 76% of their original carbon-14. Estimate the age of the Dead Sea Scrolls using the equation below.

$$A = A_0 e^{-0.000121t}$$
Example 8

An artifact originally had 16 grams of carbon-14 present. Use the decay model \( A = 16e^{-0.000121t} \) to determine how many grams of carbon-14 will be present in 6018 years.
Example 9

A bird species in danger of extinction has a population that is decreasing exponentially. Five years ago the population was at 1400 and today only 1000 of the birds are alive. Once the population drops below 100, the situation is irreversible. When will this happen?
Questions???

Make sure to be working in MyMathLab!!!