Brief Review of 2.3
What is a Polynomial Function?

**Definition of a Polynomial Function**

Let \( n \) be a nonnegative integer and let \( a_n, a_{n-1}, \ldots, a_2, a_1, a_0 \) be real numbers, with \( a_n \neq 0 \). The function defined by

\[
f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0
\]

is called a **polynomial function of degree** \( n \). The number \( a_n \) is called the **leading coefficient**.
Characteristics of Polynomials

- They are *smooth* and *continuous*.
\[ h(x) = 4 + 7x^4 - 5x^3 + x \]

- Degree?
- Leading coefficient?
- End behavior?
- Maximum number of \( x \)-intercepts?
- Maximum number of extrema?
\[ f(x) = -3(x - 7)^7(x + 1)^4 \]

- Degree?
- Leading coefficient?
- End behavior?
- Roots?
- Multiplicity of each?
Section 2.3

Dividing Polynomials
Finding Roots of a Polynomial

- Involves getting it into linear factors . . .

- Ex:
  \[ f(x) = x^3 - x^2 - 4x + 4 = (x - 1)(x - 2)(x + 2) \]

- This heavily relies on **factoring** polynomials which relies on **dividing** polynomials.
Think back to long division . . .

\[
24 \overline{)33482}
\]
Example 1

\[(x^2 + 14x + 45) ÷ (x + 9)\]
Example 2

\[(7 - 11x - 3x^2 + 2x^3) \div (x - 3)\]
What happens if we have missing terms?

- What happens with $1001 \div 5$?

- Write the polynomial in standard form.

- If any power is missing, use a zero to hold the place of that term.
Example 3

\[
\begin{array}{c}
2x^4 + 3x^3 - 7x - 10 \\
\hline
x^2 - 2x
\end{array}
\]
Synthetic Division

• Long division is long and cumbersome.

• Synthetic division is a streamlined process that works really well if you are dividing by a linear factor (e.g. \( x + 7 \)).

• Can be used for things like . . .

\[
(3x^5 + x^2 - 2x + 4) \div (x + 1)
\]
Example 4

Use synthetic division to divide $x^3 - 7x - 6$ by $x + 2$. 
Theorems

- **The Remainder Theorem**
  
  If the polynomial $f(x)$ is divided by $x - c$, then the remainder is the value $f(c)$.

  \[ f(x) = (x - c)q(x) + r \]

- **Factor Theorem**
  
  For the polynomial $f(x)$, if $f(c) = 0$, then $x - c$ is a factor of $f(x)$. 
Example 5

Given \( f(x) = 3x^3 + 4x^2 - 5x + 3 \), use the remainder theorem to find \( f(-4) \).
Example 6

Determine if \(-1\) is a zero of
\[ g(x) = x^4 - 6x^3 + x^2 + 24x - 20. \]
Example 7

Solve the equation \(15x^3 + 14x^2 - 3x - 2 = 0\), given that \(-1\) is a zero.
Example 8

Solve the equation $x^3 - 9x^2 + 20x - 12 = 0$, given that 6 is a zero.
Questions???

Be working steadily in MyMathLab.