Section 1.8
Inverse Functions
### Definition (p. 37)

A function $g$ is the **inverse function** of the function $f$ if

$$f(g(x)) = x \text{ for all } x \text{ in the domain of } g$$

and

$$g(f(x)) = x \text{ for all } x \text{ in the domain of } f.$$  

The function $g$ is denoted by $f^{-1}$ (read “$f$ inverse”).

### Note

$f^{-1}$ does NOT mean “$f$ raised to the power of negative one.”
What is an inverse function?

- A function that “undoes” the original function.

- The domain of $f$ is equal to the range of $f^{-1}$ and vice versa.

$$f (f^{-1}(x)) = f^{-1}(f(x)) = x$$
Example 1

Show that the functions are inverses of each other.

\[ f(x) = 5x^5 - 3 \quad \text{and} \quad g(x) = \sqrt[5]{\frac{x+3}{5}} \]
Example 2

Show that the functions are inverses of each other.

\[ f(x) = \frac{5}{x-2} \text{ and } g(x) = \frac{5}{x} + 2 \]
Example 3

Show that the functions are inverses of each other.

\[ f(x) = 4x - 7 \text{ and } g(x) = \frac{x+7}{4} \]
### Additional Properties of Inverses

- $f$ and $f^{-1}$ are symmetric about the line $y = x$.

- The domain of $f$ is the range of $f^{-1}$ and the range of $f$ is the domain of $f^{-1}$.

- i.e. if $(a, b)$ is a point on the graph of $f$, then $(b, a)$ is a point on the graph of $f^{-1}$.

### The Existence of an Inverse Function (p. 39)

- A function has an inverse function if and only if it is one-to-one.

- Thus, graphically it will have to pass the horizontal line test.
Horizontal Line Test

(a) Example of a function that passes the horizontal line test.

(b) Example of a function that does not pass the horizontal line test.

(c) Example of a function that passes the horizontal line test at multiple points.

(d) Example of a function that passes the horizontal line test at a single point.
Example 4

Is the function invertible?
Example 5

Is the function invertible?
Example 6
Is the function invertible?
Example 7

Is the function invertible?
Example 8
Is the function invertible?
Example 9

Graph the inverse of the given function.
Example 10

Graph the inverse of the given function.
Example 11

Graph the inverse of

\[ y = \frac{1}{x} \]
Calculating the Inverse of a Function

To find the inverse of a given function $f$ . . .

1. Determine whether or not the function is invertible.

2. Switch the $x$’s and $y$’s.

3. Solve for $y$ and rename it $f^{-1}$. 
Example 12

Find the inverse of $f(x) = 2x + 7$. 
Example 13

Find the inverse of $f(x) = 4x^3 - 1$. 
Example 14

Find the inverse of $f(x) = \frac{3}{x} - 1$. 
Example 15

Find the inverse of $f(x) = (x + 8)^3$. 
Example 16

Find the inverse of \( f(x) = \frac{2x+1}{x-4} \).
Example 17

Find the inverse of \( f(x) = \sqrt{x + 4} \).
Questions???

Make sure to be working in MyMathLab!!!