1.2 Basics of Functions and Their Graphs

Objective 1: Find the domain and range of a relation.

A relation is any set of ordered pairs. The set of all first components of the ordered pairs is called the domain of the relation and the set of all second components is called the range of the relation.

Find the domain and range of the relation:

\{(oranges,3),(apples,2),(pears,5)\}

Domain: \{oranges, apples, pears\}  Range: \{3, 2, 5\}

Objective 2: Determine whether a relation is a function.

A function is a relation in which each member of the domain corresponds to exactly one member of the range.

Determine whether each relation is a function. Give the domain and range for each relation.

1. \{(4, 5), (6, 7), (8, 8)\}

Each member of the domain corresponds to exactly one member of the range, so the relation is a function.

The domain is \{} and the range is \{}
2. \{(5, 6), (5, 7), (6, 6), (6,7)\}

This relation is not a function since the ordered pairs (5, 6) and (5, 7) have the same first component, but different second components. Or because the ordered pairs (6, 6) and (6, 7) have the same first component, but different second components. Either one of those cases make the relation not a function.

The domain:
The range:

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A relation in which each member of the domain corresponds to exactly one member of the range is a function. Notice that more than one element in the domain can correspond to the same element in the range. Aerobics and tennis both burn 505 calories per hour.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Calories Burned per hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sitting</td>
<td>80</td>
</tr>
<tr>
<td>Walking</td>
<td>325</td>
</tr>
<tr>
<td>Aerobics</td>
<td>505</td>
</tr>
<tr>
<td>Tennis</td>
<td>505</td>
</tr>
<tr>
<td>Running</td>
<td>720</td>
</tr>
<tr>
<td>Swimming</td>
<td>790</td>
</tr>
</tbody>
</table>

Aerobics and tennis both burn 505 calories per hour.
Is this a function? Does each member of the domain correspond to precisely one member of the range? This relation is not a function because there is a member of the domain that corresponds to two members of the range. 505 corresponds to aerobics and tennis.

Objective 3: Determine whether an equation represents a function.

Functions are usually written as equations rather than as ordered pairs.
Here is an equation that models paid vacation days each year as a function of years working for the company.

\[ y = -0.016x^2 + 0.93x + 8.5 \]

The variable \( x \) represents years working for a company. The variable \( y \) represents the average number of vacation days each year. The variable \( y \) is a function of the variable \( x \). For each value of \( x \), there is one and only one value of \( y \). The variable \( x \) is called the independent variable because it can be assigned any value from the domain. Thus, \( x \) can be assigned any positive integer representing the number of years working for a company. The variable \( y \) is called the dependent variable because its value depends on \( x \). Paid vacation days depend on years working for a company.

Not every set of ordered pairs defines a function. Not all equations with the variables \( x \) and \( y \) define a function. If an equation is solved for \( y \) and more than one value of \( y \) can be obtained for a given \( x \), then the equation does not define \( y \) as a function of \( x \). So the equation is not a function.
Determine whether each equation defines $y$ as a function of $x$.

$x + y = 25$

$y = 25 - x$

Only one value of $y$ can be obtained for each value of $x$, so $y$ is a function of $x$.

$x^2 + y = 25$

$y = 25 - x^2$

Only one value of $y$ can be obtained for each value of $x$, so $y$ is a function of $x$. 
\[ x^2 + y^2 = 25 \]
\[ y^2 = 25 - x^2 \]
\[ y = \pm \sqrt{25 - x^2} \]

So if \( x = 0 \), \( y = +5 \) or \( y = -5 \)

Two values for \( y \) can be obtained for one value of \( x \), so \( y \) is not a function of \( x \).

Objective 4: Evaluate a function.

Function Notation

When an equation represents a function, it is often written using function notation such as \( f(x) \), \( g(x) \), etc. in the place of \( y \). Note: \( f(x) \) does not mean “\( f \) times \( x \)”.
\( f(x) \) is read “\( f \) of \( x \)”.
Evaluating a Function

If \( f(x) = x^2 - 10x - 3 \), evaluate each of the following and simplify:

a. \( f(-1) \)  
b. \( f(x + 2) \)  
c. \( f(-x) \)  
d. \( f(x+h) \)

Substitute whatever is in parentheses for \( x \) in the given equation.

a. \( f(-1) = \)

b. \( f(x + 2) = \)

c. \( f(-x) = \)

d. \( f(x+h) = \)

Objective 5: Use the vertical line test to identify functions.

If a relation is a function, no value of \( x \) can be paired with two or more different values of \( y \). So if a graph has two or more different points with the same first coordinate, the graph cannot represent a function.

**The Vertical Line Test for Functions**

If any vertical line intersects a graph in more than one point, the graph does not define \( y \) as a function of \( x \).
Use the vertical line test to identify graphs in which \( y \) is a function of \( x \).
Objective 6: Obtain information about a function from its graph.

You can obtain information about a function from its graph. At the right or left of a graph you will find closed dots, open dots or arrows.

A closed dot indicates that the graph does not extend beyond this point, and the point belongs to the graph.

An open dot indicates that the graph does not extend beyond this point and the point does not belong to the graph.

An arrow indicates that the graph extends indefinitely in the direction in which the arrow points.

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**Analyze the graph.**

a. Is this a function?
b. Find f(4)
c. Find f(1)
d. For what value of x is f(x)=-4
Objective 7: Identify the domain and range of a function from its graph.

Identify the function's domain and range from the graph

Domain: Range: Domain: Range:

Identify the Domain and Range from the graph.

Domain: Range:
Identify the Domain and Range from the graph.

Domain: Range:

Identify the Domain and Range from the graph.

Domain: Range:
Objective 8: Identify intercepts from a function’s graph.

We can identify x and y intercepts from a function's graph. To find the x-intercepts, look for the points at which the graph crosses the x axis. The y-intercepts are the points where the graph crosses the y axis. The zeros of a function, f, are the x values for which f(x)=0. These are the x intercepts.

By definition of a function, for each value of x we can have at most one value for y. What does this mean in terms of intercepts? A function can have more than one x-intercept but at most one y intercept.

Find the x intercept(s). Find f(-4)

x-intercept/s = f(-4) =
Find the y intercept. Find $f(2)$

$y$-intercept = $f(2) = $

Find the x and y intercepts. Find $f(3)$.

$x$-intercept = $y$-intercept = $f(3) = $

Find the domain and the range.

Domain: Range: