SECTION 5.5
Integration by Substitution

The Chain Rule & Integration

• How do we handle something like this?

\[ \int 2x(x^2 + 1)^4 \, dx \]

• Think back to how we took the derivative of a composite function like . . .

\[ \frac{d}{dx} \left[ \frac{1}{5} (x^2 + 1)^5 \right] \]
u-Substitution

**THEOREM 5.13 ANTIDIFFERENTIATION OF A COMPOSITE FUNCTION**

Let $g$ be a function whose range is an interval $I$, and let $f$ be a function that is continuous on $I$. If $g$ is differentiable on its domain and $F$ is an antiderivative of $f$ on $I$, then

$$\int f(g(x))g'(x) \, dx = F(g(x)) + C.$$  

Letting $u = g(x)$ gives $du = g'(x) \, dx$ and

$$\int f(u) \, du = F(u) + C.$$  

**Example 1**

Find the indefinite integral and check the result by differentiation.

$$\int 2x(x^2 + 1)^4 \, dx$$
Example 2
Find the indefinite integral and check the result by differentiation.

$$\int 3\sqrt{3 - 4x^2}(-8x) \, dx$$

Example 3
Find the indefinite integral and check the result by differentiation.

$$\int x^2(x^3 + 5)^4 \, dx$$
Example 4
Find the indefinite integral and check the result by differentiation.
\[ \int \frac{x^2}{(16 - x^3)^2} \, dx \]

Example 5
Find the indefinite integral and check the result by differentiation.
\[ \int 4x^3 \sin x^4 \, dx \]
Example 6
Find the indefinite integral and check the result by differentiation.

\[ \int e^{x/3} \, dx \]

Example 7
Find the indefinite integral and check the result by differentiation.

\[ \int (x + 1)e^{x^2+2x} \, dx \]
Example 8
Find the indefinite integral and check the result by differentiation.

\[ \int e^{\tan 2x} \sec^2 2x \, dx \]

Example 9
Find the definite integral.

\[ \int_0^{\sqrt{2}} xe^{-x^2/2} \, dx \]
Example 10

Find the definite integral.

\[ \int_{0}^{2} \frac{x}{\sqrt{1 + 2x^2}} \, dx \]

What do we get below?

- An odd function.

\[ \int_{-a}^{a} f(x) \, dx = \]
What do we get below?

- An even function.

\[ \int_{-a}^{a} f(x) \, dx = \]

Integration of Even & Odd Functions

**Theorem 5.16 Integration of Even and Odd Functions**

Let \( f \) be integrable on the closed interval \([-a, a]\).

1. If \( f \) is an even function, then

   \[ \int_{-a}^{a} f(x) \, dx = 2 \int_{0}^{a} f(x) \, dx. \]

2. If \( f \) is an odd function, then

   \[ \int_{-a}^{a} f(x) \, dx = 0. \]

Ex. \( \int_{-3\pi}^{3\pi} \sin x \, dx = \)

Ex. \( \int_{-\pi}^{\pi} \cos x \, dx = \)
Thinking Problem

\[ \int x\sqrt{x} + 1 \, dx \]