Section 6.5
Complex Numbers in Polar Form

Complex Plane

\[ b \]
\[ \bullet \quad z = a + bi \]

Complex number
\[ z = a + bi \]

Plotting Complex Numbers & Finding the Absolute Value of Complex Numbers

The distance from 0 to a number \( a \) on a number line is \(|a|\).

The distance from the origin to the point \( z \) in the complex plane is the absolute value of \( z = a + bi \) denoted by \(|z|\)

\[ |z| = \sqrt{a^2 + b^2} \]
Plot each complex number and find its absolute value.

\[ z = 3i \]
Same as
\[ z = 0 + 3i \]

\[ z = 4 \]
\[ z = 4 + 0i \]

\[ z = 2 + 5i \]
Polar Form of a Complex Number

$z = a + bi$ is in rectangular form.

$b \bullet (a, b)$

$$r = \sqrt{a^2 + b^2}$$

$$\cos \theta = \frac{a}{r} \quad \sin \theta = \frac{b}{r} \quad \tan \theta = \frac{b}{a}$$

$$a = r \cos \theta \quad b = r \sin \theta$$

$z = a + bi$

$Z = r \cos \theta + (r \sin \theta)i$

$Z = r (\cos \theta + i \sin \theta)$ This is the polar form of a complex number.
The value of $r$ is called the modulus of $z$ and the angle $\theta$ is called the argument of $z$ with $0 \leq \theta < 2\pi$. Do not forget the interval in which $\theta$ must lie when writing in polar form.

Plot each complex number. Then write the complex number in polar form. Argument may be expressed in degrees or radians.

\[
1 + \sqrt{3} \, i
\]

\[
a = 1
\]

\[
b = \sqrt{3}
\]

\[
\text{Find } r \quad r = \sqrt{(1)^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = \sqrt{4} = 2
\]

\[
\text{Find } \theta \\
\tan \theta = \frac{b}{a} = \frac{\sqrt{3}}{1} = \sqrt{3}
\]

Since $\theta$ is in quadrant I, $\theta = \pi/3$

\[
z = 1 + \sqrt{3} \, i = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})
\]

or $2(\cos 60^\circ + i \sin 60^\circ)$
Change from polar form to rectangular form or writing a complex number in rectangular form.

Write each complex number in rectangular form. Round to nearest tenth.

$12(\cos 60^\circ + i\sin 60^\circ)$
$4(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$

$30(\cos 2.3 + i \sin 2.3)$
How to Find the Product of Two Complex Numbers

\[ z_1 = r_1 (\cos \theta_1 + i \sin \theta_1) \]
\[ z_2 = r_2 (\cos \theta_2 + i \sin \theta_2) \]
\[ z_1 z_2 = r_1 r_2 [\cos (\theta_1 + \theta_2) + i \sin (\theta_1 + \theta_2)] \]

Find products. Leave answers in polar form.

\[ z_1 = 4 (\cos 15^\circ + i \sin 15^\circ) \]
\[ z_2 = 7 (\cos 25^\circ + i \sin 25^\circ) \]

\[ z_1 = 3 (\cos 120^\circ + i \sin 120^\circ) \]
\[ z_2 = 6 (\cos 250^\circ + i \sin 250^\circ) \]

Find \( z_1 z_2 \)
How to Find the Quotient of Two Complex Numbers

\[ \frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)] \]

\[ z_1 = 50(\cos 80^\circ + i \sin 80^\circ) \]
\[ z_2 = 10(\cos 20^\circ + i \sin 20^\circ) \]

Power of a Complex Number