Section 5.1
Verifying Trig Identities

Fundamental Trigonometric Identities

Reciprocal Identities
\[
\begin{align*}
\sin x &= \frac{1}{\csc x} & \cos x &= \frac{1}{\sec x} & \tan x &= \frac{1}{\cot x} \\
\csc x &= \frac{1}{\sin x} & \sec x &= \frac{1}{\cos x} & \cot x &= \frac{1}{\tan x}
\end{align*}
\]

Quotient Identities
\[
\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}
\]

Pythagorean Identities
\[
\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x
\]

Even-Odd Identities
\[
\begin{align*}
\sin(-x) &= -\sin x & \cos(-x) &= \cos x & \tan(-x) &= -\tan x \\
\csc(-x) &= -\csc x & \sec(-x) &= \sec x & \cot(-x) &= -\cot x
\end{align*}
\]

Study Tip

Memorize the identities in the box. You may need to use variations of these fundamental identities. For example, instead of \( \sin^2 x + \cos^2 x = 1 \), you might want to use
\[
\sin^2 x = 1 - \cos^2 x
\]
or
\[
\cos^2 x = 1 - \sin^2 x.
\]

Therefore, it is important to know each relationship well so that mental algebraic manipulation is possible.
Using Fundamental Identities to Verify Other Identities

**Study Tip**

Verifying that an equation is an identity is different from solving an equation. You do not verify an identity by adding, subtracting, multiplying, or dividing each side by the same expression. If you do this, you have already assumed that the given statement is true. You do not know that it is true until after you have verified it.

When proving identities, be sure to write the variable associated with each trigonometric function. Do not get lazy and write 

\[ \sin \tan + \cos \]

for

\[ \sin x \tan x + \cos x \]

because \( \sin, \tan, \) and \( \cos \) are meaningless without specified variables.

**Guidelines for Verifying Trigonometric Identities**

- Work with each side of the equation independently of the other side. Start with the more complicated side and transform it in a step-by-step fashion until it looks exactly like the other side.
- Analyze the identity and look for opportunities to apply the fundamental identities.
- Try using one or more of the following techniques:
  1. Rewrite the more complicated side in terms of sines and cosines.
  2. Factor out the greatest common factor.
  3. Separate a single-term quotient into two terms:
     \[ \frac{a + b}{c} = \frac{a}{c} + \frac{b}{c} \]
     \[ \frac{a - b}{c} = \frac{a}{c} - \frac{b}{c} \]
  4. Combine fractional expressions using the least common denominator.
  5. Multiply the numerator and the denominator by a binomial factor that appears on the other side of the identity.
- Don’t be afraid to stop and start over again if you are not getting anywhere. Creative puzzle solvers know that strategies leading to dead ends often provide good problem-solving ideas.
Verify each of the following identities by simplifying one side only.

\[ \cos x \csc x = \cot x \]

\[ \cot x \sec x \sin x = 1 \]
\[ \cos^2 x - \sin^2 x = 2\cos^2 x - 1 \]

\[ \csc x - \csc x \cos^2 x = \sin x \]
\[
tan \theta + cot \theta = sec \theta csc \theta
\]

\[
\frac{\cos \theta \sec \theta}{\cot \theta} = \tan \theta
\]
\[ \cos^2 \theta (1 + \tan^2 \theta) = 1 \]

\[ \cos t \cot t = \frac{1 - \sin^2 t}{\sin t} \]
\[
\frac{\sec^2 t}{\tan t} = \sec t \csc t
\]

\[
\frac{\sin x}{\cos x + 1} + \frac{\cos x - 1}{\sin x} = 0
\]
\[ 1 - \frac{\cos^2 x}{1 + \sin x} = \sin x \]