4.8 Applications of Trig Functions

Solving Right Triangles. Round angles to the nearest tenth of a degree, and lengths to two decimal places.

\[ A = 41.5^\circ, \ b = 20 \]

\[
\begin{align*}
A &= 41.5^\circ & a = \\
B &= & b = 20 \\
C &= 90^\circ & c =
\end{align*}
\]

\[ b = 4, \ c = 9 \]

\[
\begin{align*}
A &= & a = \\
B &= & b = 4 \\
C &= 90^\circ & c = 9
\end{align*}
\]
Bearings
The bearing from point O to point P is the acute angle, measured in degrees, between ray OP and a north-south line.
Use the figure to solve the following.

Find the bearing from O to A.  Find the bearing from O to C.
Find the bearing from O to B.  Find the bearing from O to D.

Simple Harmonic Motion

A ball is attached to a spring hung from the ceiling. You pull the ball down 4 inches and then release it. If you neglect the effects of friction and air resistance, the ball will continue bobbing up and down on the end of the spring. These up-and-down oscillations are called simple harmonic motion.

We will use a $d$-axis, where $d$ represents distance. On this axis, the position of the ball before you pull it down is $d = 0$. This rest position is called the equilibrium position. Now you pull the ball down 4 inches to $d = -4$ and release it.
Simple Harmonic Motion

An object that moves on a coordinate axis is in simple harmonic motion if its distance from the origin, $d$, at time $t$ is given by either

$$d = a \cos \omega t$$

or

$$d = a \sin \omega t.$$  

The motion has amplitude $|a|$, the maximum displacement of the object from its rest position. The period of the motion is $2\pi/\omega$, where $\omega > 0$. The period gives the time it takes for the motion to go through one complete cycle.

In describing simple harmonic motion, the equation with the cosine function, $d = a \cos \omega t$, is used if the object is at its greatest distance from rest position, the origin, at $t = 0$. By contrast, the equation with the sine function, $d = a \sin \omega t$, is used if the object is at its rest position, the origin at $t = 0$.

**Frequency of an Object in Simple Harmonic Motion**

An object in simple harmonic motion given by

$$d = a \cos \omega t \quad \text{or} \quad d = a \sin \omega t$$

Has frequency $f$ given by $\omega/2\pi$, $\omega > 0$. Equivalently, $f = 1/\text{period.}$
An object is attached to a coiled spring. The object is pulled down (negative direction from the rest position) and then released. The object is propelled downward from its rest position at time $t = 0$. Write an equation for the distance of the object from its rest position after $t$ seconds.

When the object is released $t = 0$ and $d$ (the distance from the rest position) is 8 inches down. Since it is down, $d$, is negative. This means $t = 0$ and $d = -8$. The greatest distance from rest position occurs at $t=0$, so we use $y = a \cos \omega t$. $|a|$ is the maximum distance. Since the object initially moves down, $a = -8$.

We can find $\omega$ by using the formula for the period.  
Period = $2\pi/\omega = 2$  
$2\omega = 2\pi$  
$\omega = 2\pi/2 = \pi$  
Substitute into $d = a \cos \omega t$  
$d = -8 \cos \pi t$ (the equation for the object's simple harmonic motion)
An object moves in simple harmonic motion described by the given equation, where \( t \) is measured in seconds and \( d \) in inches. In each exercise, find the following:

a. the maximum displacement  
b. the frequency  
c. the time required for one cycle.

24. \( d = -8 \cos \left( \frac{\pi}{2} t \right) \)

a. The maximum displacement is the amplitude.  
   \( a = -8 \), so max displacement is 8
b. The frequency is \( f = \frac{\omega}{2\pi} = \frac{\pi/2}{2\pi} = \frac{1}{4} \)  
   \( \frac{1}{4} \) inch per second  
c. period = \( \frac{2\pi}{\omega} = \frac{2\pi}{\pi/2} = 2\pi(2/\pi) = 4 \)
d. Time required for one cycle is 4 seconds.

Find the length \( x \) to the nearest whole number.