Section 7.3
Complex Fractions

A complex fraction is one with a fraction in the numerator, or the denominator, or both.

Simplifying a Complex Fraction
Method 1
Simplify the numerator so that it is one fraction and the denominator so that it is one fraction.
Then multiply the numerator by the reciprocal of the denominator.
Simplify the resulting fraction, if possible.
Simplifying Complex Fractions (Method 1)

Use Method 1 to simplify the complex fraction.

\[
\frac{y + 2}{\frac{y}{y - 2}} \div \frac{\frac{y - 2}{3y}}{}
\]

Both the numerator and denominator are already simplified.

**Solution:**

\[
= \frac{y + 2}{y} \div \frac{y - 2}{3y}
\]

Write as a division problem.

\[
= \frac{y + 2}{y} \cdot \frac{3y}{y - 2}
\]

Multiply by the reciprocal.

\[
= \frac{3(y + 2)}{y - 2}
\]

Multiply.

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Simplifying Complex Fractions (Method 1) (cont'd)

Use Method 1 to simplify the complex fraction.

\[
\frac{4 - \frac{3}{x}}{5 - \frac{1}{x}}
\]

Simplify the numerator and denominator.

**Solution:**

\[
= \frac{4 \left( \frac{x}{x} \right) - \frac{3}{x}}{5 \left( \frac{x}{x} \right) - \frac{1}{x}}
\]

\[
= \frac{4x - 3}{5x - 1}
\]

\[
= \frac{\frac{x}{5x - 1}}{\frac{x}{x}}
\]

\[
= \frac{4x - 3}{5x - 1}
\]

\[
= \frac{4x - 3}{x} \cdot \frac{1}{5x - 1}
\]

\[
= \frac{4x - 3}{5x - 1}
\]
\[ \frac{24}{t+4} + \frac{6}{t} \]

\[ \frac{3y^2x^3}{8} + \frac{9y^3x^4}{16} \]

\[ \frac{2}{k} - 1 \]

\[ \frac{2}{k} + 1 \]

\[ \frac{4}{t} \cdot \frac{4}{s} \]

\[ \frac{4}{t} + \frac{4}{s} \]
\[
\frac{p - \frac{p+2}{4}}{\frac{3}{4} - \frac{5}{2p}}
\]

\[
\frac{\frac{y + 3}{y} - \frac{4}{y - 1}}{\frac{y}{y - 1} + \frac{1}{y}}
\]
Method 2
Multiply each term of the numerator and each term of the denominator by the least common denominator of all the fractions of the complex fraction.

Simplifying Complex Fractions (Method 2)

Use Method 2 to simplify the complex fraction.

\[
\frac{4 - \frac{3}{x}}{5 - \frac{1}{x}}
\]

The LCD is \(x\). Multiply the numerator and denominator by \(x\).

Solution:

\[
\left( \frac{4 - \frac{3}{x}}{5 - \frac{1}{x}} \right) \cdot x = \frac{4 \cdot x \cdot \frac{3}{x}}{5 \cdot x \cdot \frac{1}{x}} = \frac{4x - 3}{5x - 1}
\]
Simplifying Complex Fractions (Method 2) (cont’d)

Use Method 2 to simplify the complex fraction.

\[
\frac{3y + \frac{4}{y+1}}{2y - \frac{3}{y}}
\]

Multiply the numerator and denominator by the LCD \(y(y + 1)\).

Solution:

\[
\left( \frac{3y + \frac{4}{y+1}}{2y - \frac{3}{y}} \right) \cdot y(y + 1) = \frac{3y[y(y+1)] + \frac{4}{y+1} \cdot y(y+1)}{2y[y(y+1)] - \frac{3}{y} \cdot y(y+1)}
\]

\[
= \frac{3y^2(y+1) + 4y}{2y^2(y+1) - 3(y+1)}
\]

\[
\frac{2}{s - \frac{3}{t}} \cdot \frac{4t^2 - 9s^2}{st} = \frac{3y^3 + 3y^2 + 4y}{2y^5 - 2y^3 - 3y - 3}
\]

\[
\frac{1}{x^2} - \frac{1}{y^2} = \frac{1}{x} + \frac{1}{y}
\]
Simplify the expression, using only positive exponents in the answer.

\[
\frac{a^{-2} + b^{-1}}{a^{-1} - 5b^{-3}}
\]

Solution:

\[
\frac{1}{a^2} + \frac{1}{b} = \frac{1}{a} - \frac{5}{b^3}
\]

\[
\text{LCD} = a^2b^3
\]

\[
= \frac{a^2b^3}{a^2} \cdot \frac{1}{a} + \frac{a^2b^3}{b} \cdot \frac{1}{b} = \frac{b^3 + a^2b^2}{ab^3 - 5a^2}
\]
Simplifying Rational Expressions with Negative Exponents (cont’d)

Simplify the expression, using only positive exponents in the answer.

\[ \frac{x^{-3} + 2y^{-1}}{y + 2x^{3}} \]

**Solution:**

\[ \frac{1}{x^{3}} + \frac{2}{y} \]

\[ = \frac{x^{3}}{x^{3}y} + \frac{2y^{3}}{y + 2x^{3}} \]

\[ = \frac{1 \cdot y + 2 \cdot x^{3}}{x^{3}y + 2x^{3}} \]

Write with positive exponents.

\[ \frac{y + 2x^{3}}{y^{2} + 2x^{3}} \]

\[ = \frac{1}{x^{3}y} \]

\[ \frac{y + 2x^{3}}{y + 2x^{3}} \]

\[ = \frac{1}{x^{3}y} \]

**LCD = \( x^{3}y \)**

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\[ \frac{x^{-1} - y^{-1}}{x^{-2} - y^{-2}} \]
\[
\frac{a^2 - 4b^2}{3b - 6a}
\]