Section 6.5

Solving Equations by Factoring

You need to know and understand the zero-factor property.

Zero-Factor Property: If two numbers have a product of 0, then at least one of the numbers must be 0. That is, if \( ab = 0 \), then either \( a = 0 \) or \( b = 0 \).

Note: This is true only when the product is 0.
Quadratic Equation

An equation that can be written in the form

\[ ax^2 + bx + c = 0 \]

where \( a, b, \) and \( c \) are real numbers, with \( a \neq 0 \), is a \textbf{quadratic equation}. This form is called \textbf{standard form}.

Solving Quadratic Equations

1) Write the equation in standard form.
   \[ ax^2 + bx + c = 0 \]

2) Factor the polynomial.

3) Use the zero-factor property.

4) Find the solutions.

5) Check
(x + 7)(x + 3) = 0

(2q + 5)(3q − 4) = 0

x^2 + x − 12 = 0

Solve a Quadratic Equation with a Missing Linear Term

Solve 5x^2 − 80 = 0

Solution:

5x^2 − 80 = 5(x^2 − 16)
= 5(x − 4)(x + 4) Factor out 5.

x − 4 = 0 or x + 4 = 0 Solve.

x = 4 or x = −4

Check that the solution set is {−4, 4}. 
Solving an Equation That Requires Rewriting

Solve \((x + 6)(x - 2) = 2 + x - 10\).

**Solution:**

\[x^2 + 4x - 12 = x - 8\]  
**Multiply.**

\[x^2 + 3x - 4 = 0\]  
**Standard form.**

\[(x - 1)(x + 4) = 0\]  
**Factor.**

\[x - 1 = 0\]  \text{or}  \[x + 4 = 0\]

\[x = 1\]  \text{or}  \[x = -4\]

Check that the solution set is \((-4, 1)\).

\[2x^2 = 3 - x\]

\[12x^2 + 4x = 5\]

\[49x^2 + 14x = -1\]
\[ 4x^2 + 16x = 0 \]
\[ 9x^2 - 81 = 0 \]
\[ (x + 8)(x - 2) = -21 \]

\[ (3x + 1)(x - 3) = 2 + 3(x + 5) \]
Solving an Equation of Degree 3

Solve $3x^3 + x^2 = 4x$

Solution:

$3x^3 + x^2 - 4x = 0$  
**Standard form.**

$x(3x^2 + x - 4) = 0$  
**Factor out $n$.**

$x(3x + 4)(x - 1) = 0$

$x = 0$  
$x = \frac{-4}{3}$  
$x = 1$

Check that the solution set is $\left\{ -\frac{4}{3}, 0, 1 \right\}$.  

$6x^3 - 13x^2 - 5x = 0$

$z^3 - 6z^2 = -8z$
\[25x^3 = 64x\]

\[2p^3 + p^2 - 98p - 49 = 0\]