Factoring Trinomials

Factor trinomials when the coefficient of the quadratic term is 1.

Factoring \(x^2 + bx + c\)

**Step 1** Find pairs whose product is \(c\). Find all pairs of integers whose product is \(c\), the third term of the trinomial.

**Step 2** Find pairs whose sum is \(b\). Choose the pair whose sum is \(b\), the coefficient of the middle term.

If there are no such integers, the polynomials cannot be factored.

A polynomial that cannot be factored with integer coefficient is a prime polynomial.
Factor the trinomial.
\[ a^2 + 9a + 20 \]

**Solution:**

**Step 1** Find pairs of numbers whose product is 20.

\[
\begin{array}{c|c}
20(1) & 20 + 1 = 21 \\
-20(-1) & -20 + (-1) = -21 \\
10(2) & 10 + 2 = 12 \\
-10(-2) & -10 + (-2) = -12 \\
5(4) & 5 + 4 = 9 \\
-5(-4) & -5 + (-4) = -9 \\
\end{array}
\]

**Step 2** Write sums of those numbers.

The coefficient of the middle term is 9, so the required numbers are 5 and 4. The factored form of \[ a^2 + 9a + 20 \] is

\[ (a + 5)(a + 4). \]

**Check** \[ (a + 5)(a + 4) = a^2 + 9a + 20 \]
Recognizing a Prime Polynomial

Factor \( t^2 + 3t - 5 \).

**Solution:**

Look for two expressions whose product is \(-5\) and whose sum is \(3\). There are no such quantities. Therefore, the trinomial cannot be factored and is prime.
What if the leading coefficient is not 1?

You may use the trial and error method or the abc method, whichever you find easier. These two methods are explained on pages 328 and 329 in the textbook.

**Classroom Example 5**

Factor a Trinomial in $ax^2 + bx + c$ Form

Factor $6k^2 – 19k + 10$.

**Solution:**

The product is $6(10) = 60$. Look for two integers whose product is 60 and whose sum is $–19$. The necessary integers are $–15$ and $–4$. Write $–19k$ as $–15k – 4k$ and then factor by grouping.

$$
= 6k^2 – 15k – 4k + 10 \\
= (6k^2 – 15k) + (–4k + 10) \\
= 3k(2k – 5) – 2(2k – 5) \\
= (2k – 5)(3k – 2)
$$
Factor each trinomial.

**Solution:**

10x² + 17x + 3

By trial and error, the following are factored.

= (5x + 1)(2x + 3)

6r² + 13r – 5

= (2r + 5)(3r – 1)

Factoring Trinomials in ax² + bx + c Form
Factoring a Trinomial in \( ax^2 + bx + c \) Form (\( a < 0 \))

Factor \(-2p^2 - 5p + 12\).

**Solution:**

First factor out \(-1\), then proceed.

\[
= -1(2p^2 + 5p - 12) \\
= -1(p + 4)(2p - 3) \\
= -(p + 4)(2p - 3)
\]

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Factoring a Polynomial by Substitution

Factor \(8(z + 5)^2 - 2(z + 5) - 3\).

**Solution:**

\[
= 8x^2 - 2x - 3 \quad \text{Let } x = z + 5. \\
= (2x + 1)(4x - 3)
\]

Now replace \(x\) with \(z + 5\).

\[
= [2(z + 5) + 1][4(z + 5) - 3] \\
= (2z + 10 + 1)(4z + 20 - 3) \\
= (2z + 11)(4z + 17)
\]
Factor by substitution

$4(m - 5)^2 - 4(m - 5) - 15$

Don’t forget to make the final substitution for $m-5$.

42. $4(x - y)^2 - 23(x - y) - 6$
50. $2y^6 + 7xy^3 + 6x^2$