Section 4.1
Systems of Linear Equations in Two Variables

Solving a system of equations means finding the ordered pairs that are solutions to all of the equations in the system.

A system of linear equations will have either
1) One solution that is one ordered pair that is a solution to each equation
2) No solution which means the lines are parallel
3) Infinitely many solutions which means the equations are of the same line
Deciding whether an ordered pair is a solution

Example: $x + y = 6$

$4x - y = 14$; $(4, 2)$

Substitute 4 for $x$ and 2 for $y$ in each equation. If it checks for each equation, then $(4, 2)$ is a solution to the system.

- $x + y = 6$
  - $4 + 2 = 6$
  - $6 = 6$
- $4x - y = 14$
  - $4(4) - 2 = 14$
  - $16 - 2 = 14$
  - $14 = 14$
Solving a system of linear equations by substitution

Solve one equation for one variable in terms of the other variable.

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x + y = 6</td>
<td>4x – y = 14</td>
</tr>
</tbody>
</table>

\( x + y = 6 \) could be written \( x = 6 - y \)

4(6 - y) - y = 14
24 - 4y - y = 14
-5y = 14 - 24
-5y = -10
y = 2

Substitute 6 - y for the x in the other equation.

The answer must be written as an ordered pair.

(4, 2)

Now solve for x:

x + 2 = 6
x = 6 - 2
x = 4

\( x = 6y - 2 \)
\( x = \frac{3}{4} y \)
\[ x = 3y \\
3x - 9y = 0 \]

You can substitute \( 3y \) for the \( x \) in the second equation.

\[ 3(3y) - 9y = 0 \\
9y - 9y = 0 \\
0 = 0 \]

Infinitely many solutions since \( 0 = 0 \) is always true.

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\[ 8x + 2y = 4 \\
y = -4x \]

You can substitute \(-4x\) for the \( y \) in the first equation.

\[ 8x + 2(-4x) = 4 \\
8x - 8x = 4 \\
0 = 4 \\
\emptyset \ (\text{since } 0 \text{ never equals } 4) \]
Solving a system of equations by elimination

1. Write each equation in standard form
   \[ Ax + By = C \]
2. Make the coefficients of either the \( x \) terms or the \( y \) terms opposite. Do this by multiplying one or both equations by the numbers that will make the sum of either the \( x \)-terms or the \( y \)-terms equal 0.
3. Add the new equations.
4. Solve for one variable.
5. Substitute back into either of the original equations to find the other variable.
6. Check your answer in both original equations.

\[
\begin{align*}
6x + 5y &= -7 \\
-6x - 11y &= 1
\end{align*}
\]

Both equations are already in standard form and the \( y \)-terms are already opposite, so add the two equations together.
\[-2x + 3y = 1\]
\[-4x + y = -3\]

\[4x + 3y = 1\]
\[3x + 2y = 2\]
\[
\begin{align*}
8x + 4y &= 0 \\
4x - 2y &= 2
\end{align*}
\]

\[
\begin{align*}
x - 4y &= 2 \\
4x - 16y &= 8
\end{align*}
\]
\[
\frac{x}{5} + \frac{y}{5} = 6 \\
\frac{x}{10} + \frac{y}{3} = \frac{5}{6}
\]

\[
2x - 3y = 7 \\
-4x + 6y = 14
\]
If two equations have the same slope but different y-intercepts, they are parallel and the system has no solutions.

If two equations have the same slope and the same y-intercept, they are the same line and the system has infinitely many solutions.

Write each equation in slope-intercept form, and then tell how many solutions the system has.

\(-x + 2y = 8\)
\[4x - 8y = 1\]

\[5x = -2y + 1\]
\[10x = -4y + 2\]