Section 2.7

Absolute Value Equations & Inequalities

Use the distance definition of absolute value.

The absolute value of a number $x$, written $|x|$, is the distance from $x$ to 0 on the number line.

For example, the solutions of $|x| = 5$ are 5 and $-5$, as shown below. We need to understand the concept of absolute value in order to solve equations or inequalities involving absolute values. We solve them by solving the appropriate compound equation or inequality.
Use the distance definition of absolute value.

Solving Absolute Value Equations and Inequalities

Let $k$ be a positive real number and $p$ and $q$ be real numbers.

1. To solve $|ax + b| = k$, solve the compound equation $ax + b = k$ or $ax + b = -k$.

   The solution set is usually of the form $\{p, q\}$, which includes two numbers.

2. To solve $|ax + b| > k$, solve the compound inequality $ax + b > k$ or $ax + b < -k$.

   The solution set is of the form $(-\infty, p) \cup (q, \infty)$, which consists of two separate intervals.

3. To solve $|ax + b| < k$, solve the three-part inequality $-k < ax + b < k$.

   The solution set is of the form $(p, q)$, a single interval.

\[ |2x - 3| = 11 \]
First get the absolute value alone on one side.

\[ |x+4| + 7 = 13 \]

\[ |x+4| + 7 - 7 = 13 - 7 \]

\[ |x + 4| = 6 \]

\[ x + 4 = -6 \quad \text{or} \quad x + 4 = 6 \]

\[ -4 \quad -4 \quad -4 \quad -4 \]

\[ x = -10 \quad x = 2 \]

**Solve equations of the form \(|ax + b| = |cx + d|\).**

**Solving \(|ax + b| = |cx + d|\)**

To solve an absolute value equation of the form \(|ax + b| = |cx + d|\), solve the compound equation \(ax + b = cx + d\) or \(ax + b = -(cx + d)\).
\[ |x + 5| = |2x + 3| \]

\[ x + 5 = -(2x + 3) \text{ or } x + 5 = 2x + 3 \]

\[ x + 5 = -2x - 3 \quad x + 5 - 5 = 2x + 3 - 5 \]
\[ x + 2x = -3 - 5 \quad x = 2x - 2 \]
\[ x + 2x = -3 - 5 \quad x - 2x = -2 \]
\[ 3x = -8 \quad -x = -2 \]
\[ \frac{3x}{3} = \frac{-8}{3} \quad x = 2 \]
\[ x = \frac{-8}{3} \quad \{\frac{-8}{3}, 2\} \]

The absolute value can never be negative.

\[ |7x - 3| = -6 \quad \emptyset \]

The absolute value equals 0 only when you are taking the absolute value of 0.

\[ |5x - 3| = 0 \]
\[ 5x - 3 = 0 \]
\[ 5x = 3 \]
\[ x = \frac{3}{5} \]
Solve each equation.

\[ |x| = 14 \]

\[ |p - 5| = 13 \]

\[ |2x - 9| = 18 \]
\[ 2 - \frac{5}{2} m = 14 \]

Solve and graph.
\[ |9 - 3p| = 3 \]
Solve

$|x| + 3 = 10$

$|x + 5| - 2 = 12$

$|13w + 1| = -3$

$|6r - 2| = 0$
Absolute Value Inequalities

\[ |x| > 5 \]

\[ |x| < 5 \]

\[ |x| > 2 \]

\[ |4x + 1| \geq 21 \]
\[|5 - x| > 3\]

\[|r| < 1\]

\[|r + 5| \leq 20\]

\[|5 - x| \leq 3\]
\[|6x - 1| - 2 > 6\]

\[|8n + 4| < -4\]

\[|x + 9| > -3\]

\[|4x + 1| > 0\]
$|k - 4| + 5 \geq 4$