### 3.5 Exponential Growth and Decay

Exponential Growth and Decay Models

The mathematical model for exponential growth or decay is given by

\[ f(t) = A_0 e^{kt} \]  

or

\[ A = A_0 e^{kt} \]

- **If \( k > 0 \), the function models the amount, or size, of a growing entity.** \( A_0 \) is the original amount, or size, of the growing entity at time \( t = 0 \), \( A \) is the amount at time \( t \), and \( k \) is a constant representing the growth rate.
- **If \( k < 0 \), the function models the amount, or size, of a decaying entity.** \( A_0 \) is the original amount, or size, of the decaying entity at time \( t = 0 \), \( A \) is the amount at time \( t \), and \( k \) is a constant representing the decay rate.

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**Example**

A colony of bacteria grows according to the function \( N(t) = 100e^{0.045t} \). Where \( N \) is measured in grams and \( t \) is measured in days.

a. Determine the initial amount of bacteria.

b. What is the growth rate of the bacteria?

c. What is the population after 5 days?
d. How long will it take for the population to reach 140 grams?

e. What is the doubling time for the population?

Problem 1 from textbook.
The exponential models describe the population of the indicated country, \( A \), in millions, \( t \) years after 2006. Use these models to solve Exercises 1 – 6.

India \( A = 1095.4 e^{0.014t} \)
Iraq \( A = 26.8e^{0.027t} \)
Japan \( A = 127.5e^{0.001t} \)
Russia \( A = 142.9e^{-0.004t} \)

1. What was the population of Japan in 2006?
2. What was the population of Iraq in 2006?

3. Which country has the greatest growth rate? By what percentage is the population of that country increasing each year?

4. Which country has a decreasing population? By what percentage is the population of that country decreasing each year?

5. When will India’s population be 1238 million?

6. When will India’s population be 1416 million?
20. Use the exponential decay model for carbon-14, \( A = A_0e^{-0.000121t} \) to solve the given problem.

Skeletons were found at a construction site in San Francisco in 1989. The skeletons contained 88% of the expected amount of carbon-14 found in a living person. In 1989, how old were the skeletons?