3.2 Logarithmic Functions

Objective 1: Change from logarithmic to exponential form.

**Definition of the Logarithmic Function**

For $x > 0$ and $b > 0$, $b \neq 1$,

$$y = \log_{b} x \text{ is equivalent to } b^{y} = x.$$

The function $f(x) = \log_{b} x$ is the **logarithmic function with base** $b$.

The equations

$$y = \log_{b} x \quad \text{and} \quad b^{y} = x$$

are different ways of expressing the same thing. The first equation is in **logarithmic form** and the second equivalent equation is in **exponential form**.

Notice that a logarithm, $y$, is an **exponent**. You should learn the location of the base and exponent in each form.

**Location of Base and Exponent in Exponential and Logarithmic Forms**

<table>
<thead>
<tr>
<th>Exponential Form</th>
<th>Logarithmic Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b^{y} = x$</td>
<td>$y = \log_{b} x$</td>
</tr>
</tbody>
</table>

To change from logarithmic form to the more familiar exponential form, use the pattern;

$$y = \log_{b} x \text{ means } b^{y} = x$$

**Example**

Write each equation in the equivalent exponential form.

a. $4 = \log_{2} x$

b. $x = \log_{3} 9$
Objective 2: Change from exponential to logarithmic form.

**Example**

Write each equation in its equivalent logarithmic form.

a. \( b^4 = 16 \)

b. \( 5^2 = x \)

Objective 3: Evaluate logarithms.

**Example**

Evaluate.

a. \( \log_3 81 \)

b. \( \log_{36} 6 \)

c. \( \log_3 1 \)
Objective 4: Use basic logarithmic properties.

**Basic Logarithmic Properties Involving One**

1. \( \log_b b = 1 \) because 1 is the exponent to which \( b \) must be raised to obtain \( b \).
   \( \left( b^1 = b \right) \)

2. \( \log_b 1 = 0 \) because 0 is the exponent to which \( b \) must be raised to obtain 1.
   \( \left( b^0 = 1 \right) \)

**Examples:**
- \( \log_8 8 = 1 \)
- \( \log_6 1 = 0 \)

**Inverse Properties of Logarithms**

For \( b > 0 \) and \( b \neq 1 \),

- \( \log_b b^x = x \) (The logarithm with base \( b \) of \( b \) raised to a power equals that power.)
- \( b^{\log_b x} = x \) (\( b \) raised to the logarithm with base \( b \) of a number equals that number.)

**Examples:**
- \( \log_7 7^2 = 2 \)
- \( 5^{\log_5 8} = 8 \)

**Example**

Use the properties of logarithms to find the answers.

a. \( 3^{\log_3 15} \)

b. \( \log_2 2^3 \)

c. \( \log_9 9 \)

d. \( \log_3 \frac{1}{3} \)
Objective 5: Graph logarithmic functions.

![Graph of logarithmic functions](image)

**Characteristics of the Graphs of Logarithmic Functions of the Form** $f(x) = \log_b x$

- The x-intercept is 1. There is no y-intercept.
- The y-axis, or $x = 0$, is a vertical asymptote. As $x \to 0^+$, $\log_b x \to -\infty$ or $\infty$.
- If $b > 1$, the function is increasing. If $0 < b < 1$, the function is decreasing.
- The graph is smooth and continuous. It has no sharp corners or gaps.
Example

Use transformations to graph \( g(x) = 2 + \log_3(x - 3) \).
Start with \( \log_3 x \).
Example

Use transformations to graph \( g(x) = -\log_3 x \)

Objective 6: Find the domain of a logarithmic function.

We learned that the domain of an exponential function of the form \( f(x) = b^x \) includes all real numbers and its range is the set of positive real numbers. Because the logarithmic function reverses the domain and the range of the exponential function, the domain of a logarithmic function of the form \( f(x) = \log_b x \) is the set of all positive real numbers. In general, the domain of \( f(x) = \log_b g(x) \) consists of all \( x \) for which \( g(x) > 0 \).
Example

Find the domain of \( f(x) = \log_4(x - 5) \)

Find the domain of \( f(x) = \log(7 - x) \)

Objective 7: Use common logarithms.

The logarithmic function with base 10 is called the **common logarithmic function**.

\( f(x) = \log_{10} x \) is usually expressed as \( f(x) = \log x \).
Objective 8: Use natural logarithms.

The logarithmic function with base $e$ is called the **natural logarithmic function**.

The function $f(x) = \log_e x$ is usually expressed as $f(x) = \ln x$, read “el en of $x$”.

A calculator with an LN key can be used to evaluate natural logarithms.

The logarithmic function with base $e$ is called the natural logarithmic function. The function $f(x)=\log_e x$ is usually expressed as $f(x)=\ln x$. Like the domain of all logarithmic functions, the domain of the natural logarithmic function $f(x)=\ln x$ is the set of all positive real numbers. Thus the domain of $f(x)= \ln g(x)$ consists of all $x$ for which $g(x)>0$. 
Example

Find the domain of each function.

\( f(x) = \ln (x - 3) \)

\( f(x) = \ln x^2 \)

\( f(x) = \ln (x - 7)^2 \)
Evaluate or simplify each expression without using a calculator.

\[ \ln e \]

\[ \ln 1 \]

\[ e^{\ln 300} \]