Math 113 - Test 3A

April 10, 2012

Calculators are allowed on this portion of the test. The 4" x 6" formula card is allowed. No other books, notes, scratch paper are allowed. Give a complete solution to each problem and show all work. Partial credit will be given! All necessary steps must be shown to receive full credit.

1. (4 pts) Solve the triangle. Round the side lengths to the nearest tenth and the angles to the nearest degree.

\[ \sin 45^\circ = \frac{b}{100} \]
\[ b = \frac{100 \sin 45^\circ}{\sin 90^\circ} = \frac{100}{1} = 100 \]
\[ \sin 3^\circ = \frac{c}{100} \]
\[ c = \frac{100 \sin 3^\circ}{\sin 90^\circ} = \frac{100}{\frac{81.0}{c}} \]
\[ B = 180^\circ - 45^\circ - 27^\circ = 95^\circ \]

2. (5 pts) Determine whether the given measurements produce one triangle, two triangles, or no triangles at all. Solve any resulting triangles. Round the side lengths to the nearest tenth and the angles to the nearest degree. \( A = 65^\circ, \ a = 5, \) and \( b = 69 \)

Check one: \[ \text{One triangle} \quad \text{Two triangles} \quad \checkmark \text{No triangle} \]

\[ A = 65^\circ \quad a = 5 \quad \frac{5}{\sin 65^\circ} = \frac{69}{\sin B} \]
\[ B = 69^\circ \quad \frac{5 \sin B}{60} = \frac{69}{\frac{s}{\sin 65^\circ}} \]
\[ C = 125.1 \implies \text{no such angle} \]

3. (6 pts) Find \( h \) to the nearest tenth.

\[ \tan 10^\circ = \frac{x}{120} \quad \tan 30^\circ = \frac{4}{120} \]
\[ 120 \tan 10^\circ = x \quad 120 \tan 30^\circ = y \]
\[ 21.2 = x \quad 75.0 = y \]
\[ 75.0 = 21.2 + h \]
\[ 75.0 - 21.2 = h \]
\[ 53.8 = h \]
4. (7 pts) Determine whether the given measurements produce one triangle, two triangles, or no triangles at all. Solve any resulting triangles. Round the side lengths to the nearest tenth and the angles to the nearest degree. \( B = 32^\circ, a = 42, \) and \( b = 30 \)

Check one:

- One triangle
- Two triangles
- No triangle

\[ \begin{align*}
A & = 42 \\
B & = 32^\circ \\
C & = 30 \\

\text{If } A & = 42^\circ \\
C & = 180^\circ - 32^\circ - 48^\circ \\
C & = 100^\circ \\

\text{If } A & = 132^\circ \\
C & = 180^\circ - 32^\circ - 18^\circ \\
C & = 116^\circ
\end{align*} \]

5. (7 pts) Solve the triangle. Round the side lengths to the nearest tenth and the angles to the nearest degree. \( a = 66, \ b = 25, \ c = 45 \)

\[ \begin{align*}
A & = 139^\circ \\
B & = 14^\circ \\
C & = 2^\circ \\

\text{If } a & = 66 \\
66 & = b^2 + c^2 - 2bc \cos \theta \\
66^2 & = (25)^2 + (45)^2 - 2(25)(45) \cos A \\
66^2 - 25^2 - 45^2 & = -2(25)(45) \cos A \\
66^2 & - 25^2 - 45^2 & = \cos A \\
139.30^\circ & = A \\
139^\circ & = A \\

\frac{66}{\sin 139^\circ} & = \frac{25}{\sin B} \\
\frac{66 \sin B}{25} & = \frac{66 \sin 139^\circ}{25} \\
\frac{66 \sin B}{25} & = \frac{66 \sin 139^\circ}{25} \\
\sin B & = 45 \times \sin 139^\circ \\
B & = 14.38^\circ \\
B & = 14^\circ \\
C & = 180^\circ - 139^\circ - 14^\circ \\
C & = 27^\circ
\end{align*} \]
6. (5 pts) Find the area of the triangle. Round your answer to the nearest hundredth of a square inch.

\[ a = 8 \text{ in}, \ b = 12 \text{ in}, \ c = 6 \text{ in} \]

Heron's Formula:

\[ s = \frac{a + b + c}{2} = \frac{24}{2} = 12 \]

\[ A = \sqrt{s(s-a)(s-b)(s-c)} \]

\[ = \sqrt{12(12-8)(12-12)(12-6)} \]

\[ = \sqrt{12 \times 4 \times 12} \]

\[ = \sqrt{576} \]

\[ = 24 \text{ in}^2 \]

7. (8 pts) A plane leaves airport A and travels 560 miles to airport B at a bearing of N32°E. The plane leaves airport B and travels to airport C 320 miles away at a bearing of S72°E. Find the distance from airport A to airport C. Round your answer to the nearest mile.

\[ b^2 = a^2 + c^2 - 2ac \cos B \]

\[ b^2 = 320^2 + 560^2 - 2(320)(560) \cos 104° \]

\[ b \approx 709.017 \]

\[ 709 \text{ miles} \]
8. (5 pts) Plot and label the given points in the polar coordinate system.

9. (6 pts) For the point \( \left( 4, \frac{5\pi}{6} \right) \) and find other polar coordinates \((r, \theta)\) of the point for which:

(a) \( r > 0, \ -2\pi \leq \theta < 0 \)

\[ r = 4 \quad \theta = -\frac{7\pi}{6} \]

(b) \( r < 0, \ 0 \leq \theta < 2\pi \)

\[ r = -4 \quad \theta = \frac{11\pi}{6} \]
10. (6 pts) The polar coordinates of each point are given. Find the rectangular coordinates for each point.

(i) \( (-3, \frac{\pi}{4}) \)

\[
x = r \cos \theta = -3 \cos \frac{\pi}{4} = -3 \cdot \frac{\sqrt{2}}{2}
\]

\[
y = r \sin \theta = -3 \sin \frac{\pi}{4} = -3 \cdot \frac{\sqrt{2}}{2}
\]

\((x, y) = \left(-\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)\)

(ii) \( (3, 180^\circ) \)

\[
x = r \cos 180^\circ = 3(-1) = -3
\]

\[
y = 3 \sin 180^\circ = 3(0) = 0
\]

\((x, y) = (-3, 0)\)

11. (6 pts) The rectangular coordinates of each point are given. Find the polar coordinates for each point.

(i) \((0, -6) = (x, y)\)

\[
r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-6)^2} = \sqrt{36} = 6
\]

\[
\tan \theta = \frac{y}{x} = \frac{-6}{0} \text{ undefined}
\]

\[
\theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}
\]

\((r, \theta) = (6, \frac{3\pi}{2})\)

(ii) \((-3, 3)\)

\[
r = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = 3\sqrt{2}
\]

\[
\theta = \tan^{-1} \left( \frac{3}{-3} \right) \text{ Quadrant } 2
\]

\[
= \tan^{-1} (-1) = -\frac{\pi}{4}
\]

In Quad. 2 \( \Rightarrow \theta = \frac{3\pi}{4} \)

\((r, \theta) = (3\sqrt{2}, \frac{3\pi}{4})\)

12. (6 pts) Plot the complex number in the complex plane and write it in polar form. Express the argument in degrees and round to 2 decimals if needed. Hint – remember the "default" for writing a number in polar form.

\[z = 1 + i\sqrt{3}\]

\[
\theta = \tan^{-1} \left( \frac{\sqrt{3}}{1} \right) = \frac{\pi}{3}
\]

\[
\theta = \frac{\pi}{3} = 60^\circ
\]

\[z = 2 \left( \cos 60^\circ + i \sin 60^\circ \right)\]
13. (fg) Write the complex number in polar form. Express the argument in degrees.

\[ z = 2 \text{cis}(120^\circ) \]

\[ \frac{z}{\sqrt{2}} = \frac{2}{\sqrt{2}} \text{cis}(120^\circ) \]

14. Write the complex number in rectangular form.

\[ z = 2 \text{cis}(120^\circ) \]

\[ x = \text{Re}(z) = 2 \cos(120^\circ) = -1 \]

\[ y = \text{Im}(z) = 2 \sin(120^\circ) = \sqrt{3} \]

\[ z = x + iy = -1 + \sqrt{3}i \]
16. (6 pts) Find the quotient, \( \frac{z}{w} \). Leave your answer in polar form.

\[
\begin{align*}
z &= 3 \left( \cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8} \right) \\
w &= 10 \left( \cos \frac{\pi}{16} + i \sin \frac{\pi}{16} \right)
\end{align*}
\]

\[
\frac{z}{w} = \frac{3}{10} \left( \cos \left( \frac{5\pi}{8} - \frac{\pi}{16} \right) + i \sin \left( \frac{5\pi}{8} - \frac{\pi}{16} \right) \right)
\]

\[
= \frac{3}{10} \left( \cos \frac{9\pi}{16} + i \sin \frac{9\pi}{16} \right)
\]

\[
\frac{z}{w} = \frac{3}{10} \left( \cos \frac{9\pi}{16} + i \sin \frac{9\pi}{16} \right)
\]

17. (6 pts) Write the expression in standard form, \( a + bi \).

\[
\left( \sqrt{2} - i \right)^4
\]

\[
= \left( \sqrt{2} - i \right)^4 = 2^4
\]

\[
= \left( \sqrt{2} \right)^4 \left( \cos \left( -\frac{\pi}{4} \right) + i \sin \left( -\frac{\pi}{4} \right) \right)
\]

\[
= \left( \sqrt{2} \right)^4 \left( \cos \left( -4\pi \right) + i \sin \left( -4\pi \right) \right)
\]

\[
= \left( \sqrt{2} \right)^4 \left( \cos \left( 0 \right) + i \sin \left( 0 \right) \right)
\]

\[
= 16 \left( 1 + 0 \right)
\]

\[
= 16
\]

\[
= 16 \approx -6.89 - 5.79i
\]

\[
2 = \left( \sqrt{2} \right)^4 \left( \cos \left( 0 \right) + i \sin \left( 0 \right) \right)
\]