APPLICATIONS TO PHYSICS AND ENGINEERING:

WORK:
For a body that moves a distance \( d \) along a straight line as a result of being acted upon by a force of constant magnitude \( F \) in the direction of motion, we define the work \( W \) done by the force on the body as

\[
W = Fd
\]

Example: How much work is required to lift an object weighing 80 lb a distance of 5 ft?

\[
W = (80)(5 \text{ ft}) = 400 \text{ ft-lb.}
\]

If the force varies along the way, \( W = Fd \) has to be replaced by an integral formula.

Work done by a variable force \( F(x) \) in directing motion along the x-axis from \( a \) to \( b \) is

\[
W = \int_{a}^{b} F(x) \, dx
\]

Example: When a particle is located a distance \( x \) feet from the origin, a force of \( x^2 + 2x \) pounds acts on it. How much work is done in moving it from \( x = 1 \) to \( x = 3 \)?

\[
W = \int_{1}^{3} (x^2 + 2x) \, dx = \frac{50}{3}
\]
Lifting Problems:
This type of work problem occurs when the motion is vertical and the force is the gravitational force.

The force of gravity exerted on an object with mass $m$ is $F = mg$, where $g = 9.8 \text{ m/s}^2$ is the acceleration due to gravity near the surface of the Earth.

The work (in Joules) to lift an object of mass $m$ a vertical distance $x$ meters is

$$W = Fx = mgx$$

Example: An oil tank in the shape of a right circular cylinder, with height 40 m and radius 5 m is half full of oil. How much work is required to pump all the oil over the top of the tank?

$$W = \int_{a}^{b} \rho g A(x) D(x) \, dx$$

- $A(x) =$ cross-sectional area of the slice
- $D(x) =$ distance the slices must be lifted
- Weight of slice $= \rho g A(x) \Delta x$

$$W = \int_{20}^{40} \rho g (5)^2 \, x \, dx$$
\[ = 25 \pi \rho g \int_{20}^{40} x \, dx = 25 \pi \rho g \left[ \frac{x^2}{2} \right]_{20}^{40} = 15,000 \pi \rho g \text{ Joules} \]

Example: A bag of sand originally weighing 144 lb was lifted at a constant rate. As it rose, sand also leaked out at a constant rate. The sand was half gone by the time the bag had been lifted to 18 ft. How much work was done lifting the sand this far? (Neglect the weight of the bag and lifting equipment.)

The force (weight) of the bag is sand is changing. At what rate is the sand leaking from the bag?

72 lb had leaked from 0 to 18 ft or \(72/18 = 4 \text{ lb/ft}\)

\[ W = \int_{0}^{18} (144 - 4x) \, dx = 1944 \text{ ft-lb} \]