How can we find the area between these two curves?  

We could split the area into several sections, use subtraction and figure it out, but there is an easier way.

Consider a very thin rectangular (vertical) strip.  

What is the length of the strip?

\[ y_1 - y_2 \quad \text{or} \quad \left( 2 - x^2 \right) - (-x) \]

Since the width of the strip is a very small change in \( x \), we could call it \( dx \).
Since the strip is a long thin rectangle, the area of one strip is:

\[ \text{length} \cdot \text{width} = (2 - x^2 + x) \, dx \]

If we add the area of all (as many as possible) the strips, we get:

\[ \int_{-1}^{2} (2 - x^2 + x) \, dx \]

How are the bounds -1 and 2 found?

\[ \int_{-1}^{2} (2 - x^2 + x) \, dx \]

\[ 2x - \frac{1}{3}x^3 + \frac{1}{2}x^2 \bigg|_{-1}^{2} \]

\[ \left( 4 - \frac{8}{3} + 2 \right) - \left( -2 + \frac{1}{3} + \frac{1}{2} \right) \]

\[ 6 - \frac{8}{3} + 2 - \frac{1}{3} - \frac{1}{2} \]

\[ = \frac{27}{6} = \frac{9}{2} \]

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The formula for the area between curves is:

$$\text{Area} = \int_a^b \left[ f_1(x) - f_2(x) \right] dx$$

Example: Find the area between the curves $y = \sqrt{x}$, $y = x - 2$, and the x-axis.
If we try vertical strips, we have to integrate in two parts. Why?

What are the bounds for each integral?

\[
\int_0^2 \sqrt{x} \, dx + \int_2^4 \sqrt{x} - (x - 2) \, dx
\]

We can find the same area using only one horizontal strip.

The width of the strip is \( dy \). What is the length of the strip? \( x_1 - x_2 \)

Since the width of the strip is \( dy \), we find the length of the strip by solving for \( x \) in terms of \( y \).

\[
\begin{align*}
y &= \sqrt{x} \\
y^2 &= x \\
y &= x - 2 \\
y + 2 &= x
\end{align*}
\]
What are the bounds for the integral? 
How do we set up the integrand?

\[ y^2 = y + 2 \quad \rightarrow \quad y = -1, 2 \]

\[ \int_0^2 (y + 2) - y^2 \, dy \]

Function on the right minus the function on the left.

<table>
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<tr>
<th>length of strip</th>
<th>width of strip</th>
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\[ \int_0^2 (y + 2) - y^2 \, dy = \frac{1}{2} y^2 + 2y - \frac{1}{3} y^3 \bigg|_0^2 \]

\[ 2 + 4 - \frac{8}{3} = \frac{10}{3} \quad Ans \]
General Strategy for Area Between Curves:

1. Sketch the curves.

2. Decide on vertical or horizontal strips. (Pick whichever is easier to write formulas for the length of the strip, and/or whichever will let you integrate fewer times.)

3. Write an expression for the area of the strip.
   (If the width is $dx$, the length must be in terms of $x$. If the width is $dy$, the length must be in terms of $y$.)

4. Find the limits of integration. (If using $dx$, the limits are $x$ values; if using $dy$, the limits are $y$ values.)

5. Integrate to find area.

Find the area between the curves. 
$y = x\sin(x^2)$, $y = x^4$. You may use your graphing calculator.
\[ \int_{0}^{0.896} [x \sin(x^2) - x^4] \, dx \]

Ans. 0.037

Example: Find the area between \( x = 2y^2 \) and \( x = 4 + y^2 \)
Find the points of intersection to set the bounds and set up the integral.

\[ 4 + y^2 = 2y^2 \quad \rightarrow \quad y = -2, 2 \]

\[
\int_{-2}^{2} \left((4+y^2) - 2y^2\right) dy = \int_{-2}^{2} (4 - y^2) dy
\]

\[
= \left[ 4y - \frac{y^3}{3} \right]_{-2}^{2} = \left( 8 - \frac{8}{3} \right) - \left( -8 - \frac{-8}{3} \right) = \frac{32}{3} \quad \text{Ans}
\]