45 – 45 – 90 Triangle:

The hypotenuse is $\sqrt{2}$ times the hypotenuse.

30 – 60 – 90 Triangle:

i) The hypotenuse is twice the shorter leg.

ii) The longer leg is $\sqrt{3}$ times the shorter leg.
TRIANGLE TRIGONOMETRY

I) Right Triangle Trigonometry

Pythagorean Theorem
In any right triangle,
\[ c^2 = a^2 + b^2 \]

TRIGONOMETRIC RATIOS

\[
\sin A = \frac{\text{length of side opposite angle } A}{\text{length of hypotenuse}} = \frac{a}{c}
\]

\[
\cos A = \frac{\text{length of side adjacent to angle } A}{\text{length of hypotenuse}} = \frac{b}{c}
\]

\[
\tan A = \frac{\text{length of side opposite angle } A}{\text{length of side adjacent to angle } A} = \frac{a}{b}
\]
Example 1: Find the three trigonometric ratios for angle A in the triangle.

\[
\sin A = \frac{\text{length of side opposite angle A}}{\text{length of hypotenuse}} = \frac{a}{c} = \frac{144 \text{ m}}{156 \text{ m}} = 0.9231
\]

\[
\cos A = \frac{\text{length of side adjacent to angle A}}{\text{length of hypotenuse}} = \frac{b}{c} = \frac{60.0 \text{ m}}{156 \text{ m}} = 0.3846
\]

\[
\tan A = \frac{\text{length of side opposite angle A}}{\text{length of side adjacent to angle A}} = \frac{a}{b} = \frac{144 \text{ m}}{60 \text{ m}} = 2.400
\]

We can find the measures of angles A, B, and C by using a trig table or the second function on a calculator.

We know: \( A + B + C = 180^0 \), and \( C = 90^0 \)

Therefore, \( A + B = 90^0 \)

\[
A = \sin^{-1}\left(\frac{144 \text{ m}}{156 \text{ m}}\right) = 67.4^0
\]

\[
B = 90^0 - 67.4^0 = 22.6^0
\]
Mnemonic device for remembering sine, cosine, and tangents ratios:

**SOHCAHTOA**

\[
\begin{align*}
\text{Sine} &= \frac{\text{Opposite}}{\text{Hypotenuse}} \\
\text{Cosine} &= \frac{\text{Adjacent}}{\text{Hypotenuse}} \\
\text{Tangent} &= \frac{\text{Opposite}}{\text{Adjacent}}
\end{align*}
\]

**SOLVING RIGHT TRIANGLES**

To solve a triangle means to find the measures of the various parts of a triangle that are not given.

Examples: Draw, label, then solve the right triangle described below. C is the right angle.

1) \( B = 36.7^0, \; b = 19.2\text{m} \)

\[\begin{array}{c}
A \\
\text{b} \\
C \\
a \\
\end{array}\]

\[\begin{array}{c}
A = 53.3^0, \; c = 32.1\text{m}, \; \text{and} \; a = 25.8\text{m}\]

2) Find AC in right triangle ABC. \( a = 225 \text{ m} \), and \( A = 10.1^\circ \). C is the right angle. (Don’t use the Pythagorean Theorem.)

\[
\tan 10.1^\circ = \frac{225 \text{ m}}{b}
\]

\[
b \cdot \tan 10.1^\circ = 225 \text{ m}
\]

\[
b = \frac{225 \text{ m}}{\tan 10.1^\circ}
\]

\[
b = 1260 \text{ m}
\]

Some Useful Trigonometric Relationships

For Right Triangles:

\[
c^2 = a^2 + b^2 \quad \text{PYTHAGOREAN THEOREM}
\]

\[
\sin \alpha = \frac{a}{c} \quad \cos \alpha = \frac{b}{c} \quad \tan \alpha = \frac{a}{b}
\]

\[
\sin \beta = \frac{b}{c} \quad \cos \beta = \frac{a}{c} \quad \tan \beta = \frac{b}{a}
\]
The sine and cosine law (rule) for any arbitrarily shaped triangle:

![Diagram of triangle](image)

**law of sine**: \[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

In order to use the law of sines, you must know

a) two angles and any side, or

b) two sides and an angle opposite one of them.

---

**Example 1:** Solve the triangle if \( C = 28.0^\circ \), \( c = 46.8\text{cm} \), and \( B = 101.5^\circ \)

\[
\frac{c}{\sin C} = \frac{b}{\sin B} \quad \frac{46.8\text{cm}}{\sin 28.0^\circ} = \frac{b}{\sin 101.5^\circ}
\]

\[
b(\sin 28.0^\circ) = (\sin 101.5^\circ)(46.8\text{cm})
\]

\[
b = \frac{(\sin 101.5^\circ)(46.8\text{cm})}{\sin 28.0^\circ} = 97.7\text{ cm}
\]
A = 180° - B - C = 180° - 101.5° - 28.0° = 50.5°

To find side \(a\),

\[
\frac{c}{\sin C} = \frac{a}{\sin A}
\]

\[
\frac{46.8\text{cm}}{\sin 28.0°} = \frac{a}{\sin 50.5°}
\]

\[
a(\sin 28.0°) = (\sin 50.5°)(46.8\text{cm})
\]

\[
a = \frac{(\sin 50.5°)(46.8\text{cm})}{(\sin 28.0°)} = 76.9°
\]

The solution is \(a = 76.9\) cm, \(b = 97.7\) cm, and \(A = 50.5°\)

---

**law of cosine:**  
\[
a^2 = b^2 + c^2 - 2bccosA
\]

\[
b^2 = a^2 + c^2 - 2accosB
\]

\[
c^2 = a^2 + b^2 - 2abcosC
\]

In order to use the law of cosines you must know

a) two sides and the included angle, or
b) all three sides.
Example 2: Solve the triangle if \( A = 115.2^\circ \), \( b = 18.5 \text{ m} \), and \( c = 21.7 \text{ m} \).

To find side \( a \),
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
a^2 = (18.5 \text{ m})^2 + (21.7 \text{ m})^2 - 2(18.5 \text{ m})(21.7 \text{ m})(\cos 115.2^\circ)
\]

\( a = 34.0 \text{ m} \)

To find angle \( B \), use the law of sines (it requires less computation.)

\( B = 29.5^\circ \), \( C = 35.3^\circ \), \( a = 34.0 \text{ m} \)
Length and Coordinate Systems

Coordinate Systems are used to locate things with respect to a known origin.

Types of coordinate systems:
- Cartesian
- Cylindrical
- Spherical

To locate an object at point $A$, with respect to the origin (point $O$) of the Cartesian system, you move along the $x$-axis by $x$ amount (or steps) and then you move along the dashed line parallel to the $y$-axis by $y$ amount. Finally, you move along the dashed line parallel to the $z$-direction by $z$ amount. How would you get to point $A$ using the Cylindrical or Spherical system? Explain.