Probability
Calculating Probabilities of Events

Assigning Probabilities
- There are several ways to assign probabilities to the events of a sample space.
- Perform the experiment many times and assign probabilities based on empirical data.
- Determine the theoretical probability of the event.
- Use counting techniques to determine the probability of more complex events.

Probability of Equally Likely Outcomes
- Let $\mathcal{S}$ be a sample space consisting of $N$ equally likely outcomes. Let $E$ be any event. Then
  \[ \Pr(E) = \frac{\text{number of outcomes in } E}{N}. \]

Example: Equally Likely Outcomes
- Suppose that a cruise ship returns to the US from the Far East. Unknown to anyone, 4 of its 600 passengers have contracted a rare disease. Suppose that the Public Health Service screens 20 passengers, selected at random, to see whether the disease is present aboard ship. What is the probability that the presence of the disease will escape detection?

Complement Rule
- **Complement Rule**
  
  Let $E$ be any event, $E'$ its complement. Then
  \[ \Pr(E) = 1 - \Pr(E'). \]

Example: The Complement Rule
- A group of 5 people is to be selected at random. What is the probability that 2 or more of them have the same birthday?
- For simplicity we will ignore leap years and assume that each of the 365 days of the year are equally likely.
- Choosing a person at random is equivalent to choosing a birthday at random.
To Remember:

- For a sample space with a finite number of equally likely outcomes, the probability of an event is the number of elements in the event divided by the number of elements in the sample space.
- The probability of the complement of an event is 1 minus the probability of the event.

Example

- Five numbers are chosen at random from the whole numbers between 1 and 13, inclusive, with replacement.
  a) What is the probability that all the numbers are even?
  b) What is the probability that all the numbers are odd?
  c) What is the probability that at least one of the number is odd?

Example

- An urn contains 40 balls, some red and some white. If the probability of selecting a red ball is .45, how many red balls are in the urn?

Example

- Of the nine members of the board of trustees of a college, five agree with the president on a certain issue. The president selects three trustees at random and asks for their opinions. What is the probability that at least two of them will agree with him?

Example

- A certain mathematics classroom is made up of 12 boys and 10 girls. Seven students are asked to go to the blackboard to work a problem. What is the probability that the first three children chosen are boys?

Example

- An airport limousine has four passengers and stops at six different hotels. What is the probability that two or more people will be staying at the same hotel? (Assume that each person is just as likely to stay in one hotel as another.)
Example

- In a certain manufacturing process the probability of a type I defect is .12, the probability of a type II defect is .22, and the probability of having both types of defects is .02. Find the probability of having neither type of defect.

Example

- A bag contains nine apples, of which two are Fuji apples. A sample of three apples is selected at random. What is the probability that sample contains:
  a.) exactly one Fuji apple?
  b.) no Fuji apples?

Example

- A vacationer has brought along four novels and four nonfiction books. One day the person selects two at random to take to the beach. What is the probability that both are novels?