Solve the problem.

1) Write the equation of the line $-4x + 2y = 5$ in standard form.

2) Write the equation of the straight line that is parallel to the $y$-axis with $x$-intercept of (-2,0).

3) Find the $x$-intercept and $y$-intercept for the line $y = 2$.

4) Find the $x$-intercept and $y$-intercept for the line $y = 5x$.

5) Find the $x$-intercept and $y$-intercept for the line $y = -2x + 6$.

6) Graph the equation $2x - 5y = -10$.

7) The value $y$ of a machine (in dollars) is known to depreciate linearly with time $x$ (measured in years from the time it was bought new). Suppose that $y$ is related to $x$ by the equation $y = 2,000 - 200x$.
   (a) Sketch the graph of this linear equation for $0 \leq x \leq 10$.
   (b) What is the value of the machine when it is 5 years old?
   (c) What is the economic interpretation of the $y$-intercept of the graph?
   (d) When will the value of the machine reach the scrap value of $400$?

8) Find the point of intersection of the two lines $x + 4y = 6$ and $x - 4y = 2$.

9) Find the point of intersection of the two lines $x + 3y = 6$ and $x - y = 2$. 

Name______________________________
10) Solve the system of linear equations:
\[
\begin{align*}
5x - 3y &= 12 \\
x &= 3
\end{align*}
\]

11) Solve the system of linear equations:
\[
\begin{align*}
2x + 2y &= 1 \\
3x - y &= 6
\end{align*}
\]

12) Does (0, -4) satisfy the following system?
\[
\begin{align*}
y &= 2x - 4 \\
2y &= 7x - 8
\end{align*}
\]

13) Suppose that the supply and demand equations of a certain commodity are given by $q = 5p - 15$ and $q = -2.5p + 30$ respectively, where $p$ is the unit price of the commodity in dollars and $q$ is the quantity.
   (a) What is the supply when the price is $8$?
   (b) What is the demand when the price is $8$?
   (c) Find the equilibrium price and the corresponding number of units supplied and demanded.
   (d) Draw the graphs of the supply and demand equations on the same set of axes.
   (e) Find where the two lines cross the horizontal axis and give an economic interpretation of these points.

14) Suppose that the supply and demand equations of a new CD at a store are given by $q = 3p - 12$ and $q = -2p + 23$ respectively, where $p$ is the unit price of the CD’s in dollars and $q$ is the quantity.
   (a) What is the supply when the price is $10$?
   (b) What is the demand when the price is $10$?
   (c) Find the equilibrium price and the corresponding number of units supplied and demanded.
   (d) Find where the two lines cross the horizontal axis and give an economic interpretation of these points.

**Find the equation for the line described.**

15) The line passing through the point (2, 3) and having slope -4

16) The line through (5,2) and (-2, 2)

17) The line having y-intercept (0, 5) and parallel to $2x + y = 10$

18) The line having y-intercept (0, 2) and perpendicular to $y = -4x + 20$
19) The horizontal line passing through the point (2, -3)

20) The line perpendicular to $3x + 2y = 5$ and passing through (1, 3)

21) Perpendicular to $y = -\frac{3}{2}x + 2$ and passing through the point (0, 0).

Solve the problem.

22) Suppose a manufacturer finds that the number of units $x$ she produces and the cost $y$ of producing $x$ units are related by an equation of the form $y = mx + b$. If it costs $2300 to produce 10 units and $2450 to produce 15 units, what does it cost to produce 20 units?

23) A game company has fixed costs of $40,000 per year. Each game costs $12.00 to produce and sells for $18.00. How many games must the company produce and sell each year in order to make a profit of $95,000?

24) A salesman’s weekly pay depends on his volume of sales. He earns $80 each week in addition to $10 for each item he sells.
   (a) Write an equation relating $y$, the salesman weekly pay, to $x$, the number of items he sells.
   (b) How many items must he sell for his pay to be $300.

25) The cost to run a TV factory depends on the number of items produced. The factory’s fixed cost is $10,000 each month in addition to $120 for each TV produced.
   (a) Write an equation relating $y$, the factory’s costs, to $x$, the number of TV’s produced.
   (b) When will the factory’s costs reach $22,000?

26) A water tank is being emptied such that the height $y$ (in feet) of the water inside the tank decreases at a linear rate with time $t$ measured in hours past 12:00 noon. If the height was 15 feet at 1:00 pm and 7.5 feet at 6:00 pm, find
   (a) the equation relating $y$ to $t$,
   (b) the height at 12:00 noon, and
   (c) the total time required to empty the tank.

State whether the calculation is possible. If possible, perform the calculation.

27) \[
\begin{bmatrix}
2 & 1 & 0 \\
3 & 0 & 2 \\
\end{bmatrix}
\begin{bmatrix}
7 & -1 \\
1 & 0 \\
5 & 2 \\
\end{bmatrix}
\]

28) \[
\begin{bmatrix}
1 \\
2 \\
\end{bmatrix}
\begin{bmatrix}
4 & 0 & 3 \\
-5 \\
\end{bmatrix}
\]

29) \[
\begin{bmatrix}
1 & 0 & -2 \\
2 & 4 & 3 \\
\end{bmatrix}
\begin{bmatrix}
2 & 1 & -1 \\
5 & 0 & 1 \\
\end{bmatrix}
\]
30) \[
\begin{bmatrix}
3 & 1 \\
-1 & 0
\end{bmatrix}
\begin{bmatrix}
2 & 7 \\
6 & 4 \\
1 & 6
\end{bmatrix}
\]

Solve the problem.

31) Are the following matrices inverses of each other?
\[A = \begin{bmatrix} 1 & 4 \\ 8 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 8 \\ 3 & -2 \end{bmatrix}\]

Find the inverse of the matrix, if it exists.

32) \[
\begin{bmatrix}
3 & 1 \\
6 & 2
\end{bmatrix}
\]

33) \[
\begin{bmatrix}
2 & -1 \\
-6 & 8
\end{bmatrix}
\]

34) \[
\begin{bmatrix}
3 & 2 \\
0 & 1
\end{bmatrix}
\]

35) \[
\begin{bmatrix}
3 & 1 \\
0 & 0
\end{bmatrix}
\]

Solve the problem.

36) Solve the system \[
\begin{aligned}
x + y &= 2 \\
2x + y &= -1
\end{aligned}
\]
by using the inverse of an appropriate matrix.

37) Consider the system \[
\begin{aligned}
2x + 3y &= 4 \\
-2x - y &= 8
\end{aligned}
\]
(a) Rewrite it in the form \(AX = B\), where \(A\), \(B\), and \(X\) are appropriate matrices.
(b) Find the inverse of \(A\).
(c) Solve the system by computing \(A^{-1}B\).

38) Consider the system \[
\begin{aligned}
0.7x + 0.2y &= 3 \\
0.3x + 0.8y &= 2
\end{aligned}
\]
(a) Rewrite it in the form \(AX = B\) where, \(A\), \(B\), and \(X\) are appropriate matrices.
(b) Find the inverse of \(A\).
(c) Solve the system by computing \(A^{-1}B\).

39) (a) Find the inverse of the matrix \[
\begin{bmatrix}
2 & 1 \\
3 & 1
\end{bmatrix}
\]
(b) Use the result from (a) to solve the system \[
\begin{aligned}
2x + y &= 2 \\
3x + y &= -1
\end{aligned}
\]

Determine whether the matrix is a stochastic matrix. Give a reason for your conclusion.

40) \[
\begin{bmatrix}
\frac{1}{3} & -\frac{1}{3} \\
\frac{2}{3} & \frac{4}{3}
\end{bmatrix}
\]
41) \[
\begin{bmatrix}
0.2 & 0 & 0 \\
0 & 0.3 & 0 \\
0.8 & 0.4 & 1
\end{bmatrix}
\]

42) \[
\begin{bmatrix}
0.2 & 0.8 \\
0.8 & 0.2
\end{bmatrix}
\]

43) \[
\begin{bmatrix}
0.2 & 0 \\
0.8 & 1
\end{bmatrix}
\]

**Solve the problem.**

44) Suppose \( A = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix} \). Find \( A^2 \) and \( A^3 \).

45) If \( A = \begin{bmatrix} 0.4 & 0.8 & 0.1 \\ 0.3 & 0.1 & 0.4 \\ 0.3 & 0.1 & 0.5 \end{bmatrix} \). Find \( A^2 \).

46) Assume that the transition matrix of a Markov process is given by the following.
\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
What does this say about the change from year to year of a given initial distribution?

47) Assume 50% of women currently work. Of those who work, 70% of their daughters work the next generation. Of those who don’t work, only 40% of their daughters work the next generation. Find the percentage of women in the next generation who work.

48) A Markov process has the transition matrix
\[
A = \begin{bmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{bmatrix}
\]
and the initial distribution matrix \( \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \).
Find the first and second distribution matrices.

49) In a certain company it has been determined that in one year 10% of the skilled workers become unskilled and 40% of the unskilled workers become skilled.
(a) Write a stochastic matrix, labeling the rows and columns with \( S \) for skilled and \( U \) for unskilled, which describes the transitions.
(b) Assume that at the beginning of a year 70% of the workers are skilled and 30% of the workers are unskilled. What percentage of the workers will be unskilled at the beginning of the next year?
50) In a certain city, it has been determined that 70% of the small businesses (employing 25 or fewer employees) grow to be medium-sized businesses (26–75 employees) each year and 40% of the medium-sized businesses shrink to small businesses.
(a) Write a stochastic matrix, labeling the rows and columns with $S$ for small and $M$ for medium, which describes the transitions.
(b) Assume that at the beginning of a year, 20% of the businesses are small and 80% are medium-sized. What percentage of the businesses will be medium-sized at the beginning of the next year?

Determine whether the matrix is a regular stochastic matrix. Give a reason for your conclusion.

51) \[
\begin{pmatrix}
0.3 & 0.9 \\
0.7 & 0.1
\end{pmatrix}
\]

52) \[
\begin{pmatrix}
0.3 & 1 \\
0.7 & 0
\end{pmatrix}
\]

Find the stable distribution of the regular stochastic matrix.

53) \[
\begin{pmatrix}
0.6 & 0.5 \\
0.4 & 0.5
\end{pmatrix}
\]

54) \[
\begin{pmatrix}
0.4 & 0.5 \\
0.6 & 0.5
\end{pmatrix}
\]

Solve the problem.

55) Suppose that 100 cows graze in pasture I and 10 cows graze in pasture II. Assume that the location of cows from one day to the next can be modeled by a Markov process whose transition matrix is given by
\[
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
0.4 & 0.5 \\
0.6 & 0.5
\end{pmatrix}
\]
How many cows will be in pasture I after four days?

56) A survey in a small mining town indicates that 40% of miners’ sons become miners, whereas 60% do not. Also 10% of nonminers’ sons become miners. At the present time, 70% of the town’s men are miners and 30% are nonminers.
Considering this as a Markov process:
(a) Write the initial distribution matrix.
(b) Write the stochastic (transition) matrix with rows and column labeled $M$ (for miners) and $NM$ (for nonminers) that describes the transitions.
(c) Give the percentage of the town’s men who will become miners in the first generation, ... in the second generation.
(d) Write the system of linear equations needed to determine the stable distribution of the town’s male population.
(e) Give the proportion of the town’s men who will become miners in the long run.

57) A national study of collegiate beer drinkers revealed the following facts:
(I) Among current beer drinkers, 30% prefer one of the "light" beers and 70% prefer "regular" beer.
(II) In any given year, 20% of the "light" drinkers switch to "regular," and 25% of the "regular" drinkers switch to "light."

(a) Write the initial distribution matrix. (Label the matrix using $L$ for "light" and $R$ for "regular."
(b) Write the transition matrix for the study. (Label columns and rows using $L$ for "light" and $R$ for "regular."
(c) In the long run what proportion of collegiate beer drinkers drink "light"?
1) \( y = 2x + \frac{5}{2} \)

2) \( x = -2 \)
3) \( x \)-intercept: none
   \( y \)-intercept: \((0, 2)\)
4) \( x \)-intercept: \((0, 0)\)
   \( y \)-intercept: \((0, 0)\)
5) \( x \)-intercept: \((3, 0)\)
   \( y \)-intercept: \((0, 6)\)

6) 

7) (a)

(b) \$1000
(c) the value of the machine when it is new
(d) 8 years

8) \((4, \frac{1}{2})\)

9) \((3, 1)\)

10) \((3, 1)\)

11) \((\frac{13}{8}, -\frac{9}{8})\)

12) Yes
13) (a) 25  
(b) 10  
(c) (6, 15); The equilibrium price is $6 when 15 units are supplied and demanded.  
(d)  

\[ \begin{array}{c}
\text{supply} \\
\hline
35 \\
30 \\
25 \\
20 \\
15 \\
10 \\
5 \\
\hline
2 \\
4 \\
6 \\
8 \\
10 \\
12 \\
14 \\
\end{array} \]

\[ \begin{array}{c}
\text{demand} \\
\hline
5 \\
10 \\
15 \\
20 \\
25 \\
30 \\
35 \\
\hline
2 \\
4 \\
6 \\
8 \\
10 \\
12 \\
14 \\
\end{array} \]

(e) (3, 0), (12, 0)  
*Economic interpretation:* The intersection of the supply curve with the horizontal axis indicates the lowest price at which the manufacturer is willing to sell the product or service. The intersection of the demand curve with the horizontal axis is the highest price a consumer is willing to pay for a product or service.  

14) (a) 18  
(b) 3  
(c) (7,9); The equilibrium price is $7 when 9 units are supplied and demanded.  
(d) (4, 0), (\frac{23}{2}, 0)  

**Economic interpretation:** The intersection of the supply curve with the horizontal axis indicates the lowest price at which the store is willing to sell the CD's. The intersection of the demand curve with the horizontal axis is the highest price a consumer is willing to pay for the CD.  

15) \( y = -4x + 11 \)  
16) \( y = 2 \)  
17) \( y = -2x + 5 \)  
18) \( y = \frac{1}{4}x + 2 \)  
19) \( y = -3 \)  
20) \( y = \frac{2}{3}x + \frac{7}{3} \)  
21) \( y = \frac{2}{3}x \)  
22) $2600  
23) 22,500 games  
24) (a) \( y = 10x + 80 \)  
(b) 22  
25) (a) \( y = 120x + 10,000 \)  
(b) 100  
26) (a) \( y = -1.5t + 16.5 \)  
(b) 16.5 feet  
(c) 11 hours  
27) \[ \begin{bmatrix} 15 & -2 \\ 31 & 1 \end{bmatrix} \]
28) \[
\begin{bmatrix}
4 & 0 & 3 \\
8 & 0 & 6 \\
-20 & 0 & -15
\end{bmatrix}
\]
29) The calculation is not possible.
30) The calculation is not possible.
31) No.
32) Inverse does not exist.
33) \[
\begin{bmatrix}
0.8 & 0.1 \\
0.6 & 0.2
\end{bmatrix}
\]
34) \[
\begin{bmatrix}
1 & 2 \\
3 & 3 \\
0 & 1
\end{bmatrix}
\]
35) Inverse does not exist.
36) \(x = -3, y = 5\)
37) (a) \[
\begin{bmatrix}
2 & 3 \\
-2 & -1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix} 4 \\
8 \end{bmatrix}
\]
(b) \[
\begin{bmatrix}
-1 & -3 \\
4 & 4 \\
1 & 1 \\
2 & 2
\end{bmatrix}
\]
(c) \(x = -7, y = 6\)
38) (a) \[
\begin{bmatrix}
0.7 & 0.2 \\
0.3 & 0.8
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix} 3 \\
2 \end{bmatrix}
\]
(b) \[
\begin{bmatrix}
-1.6 & -0.4 \\
-0.6 & 1.4
\end{bmatrix}
\]
(c) \(x = 4, y = 1\)
39) (a) \[
\begin{bmatrix}
-1 & 1 \\
3 & -2
\end{bmatrix}
\]
(b) \(x = -3, y = 8\)
40) No; an entry is negative.
41) No; the second column does not sum to 1.
42) Yes; it is a square matrix with non-negative entries whose columns sum to 1.
43) Yes; this is a square matrix with nonnegative entries and each column sums to 1.
44) \(A^2 = \begin{bmatrix} 0.68 & 0.32 \\
0.32 & 0.68 \end{bmatrix}, \quad A^3 = \begin{bmatrix} 0.392 & 0.608 \\
0.608 & 0.392 \end{bmatrix}\)
45) \(A^2 = \begin{bmatrix} 0.43 & 0.41 \\
0.27 & 0.29 \\
0.3 & 0.3 \\
0.32
\end{bmatrix}\)
46) There is no change.
47) 55%
48) \[
\begin{bmatrix}
0.65 \\
0.35
\end{bmatrix}
\text{ and } \begin{bmatrix}
0.725 \\
0.275
\end{bmatrix}\]
49) (a) \[
\begin{pmatrix}
S & U \\
0.9 & 0.4 \\
0.1 & 0.6
\end{pmatrix}
\]
(b) 25%

50) (a) \[
\begin{pmatrix}
S & M \\
0.3 & 0.4 \\
0.7 & 0.6
\end{pmatrix}
\]
(b) 62%

51) Yes; the matrix is a stochastic all of whose entries are positive.

52) Yes; squaring the matrix yields \[
\begin{pmatrix}
0.79 & 0.3 \\
0.21 & 0.7
\end{pmatrix}
\], which has all positive entries.

53) \[
\begin{pmatrix}
5 \\
4 \\
9
\end{pmatrix}
\]

54) \[
\begin{pmatrix}
5 \\
11 \\
6 \\
11
\end{pmatrix}
\]

55) 50

56) (a) \[
\begin{pmatrix}
0.7 \\
0.3
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
M & NM \\
0.4 & 0.1 \\
0.6 & 0.9
\end{pmatrix}
\]
(c) 31%; 19.3%

(d) \[
\begin{cases}
x + y = 1 \\
0.4x + 0.1y = x \\
0.6x + 0.9y = y
\end{cases}
\]
(e) \[
\frac{1}{7}
\]

57) (a) \[
\begin{pmatrix}
L \\
R
\end{pmatrix}
\begin{pmatrix}
0.3 \\
0.7
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
L & R \\
0.8 & 0.25 \\
0.2 & 0.75
\end{pmatrix}
\]
(c) \[
\frac{5}{9}
\]