SKILLS NEEDED:

1) **How to Algebraically Write:**
   
   **a) Consecutive Integers**
   
   Ex) First Integer = \(a\)
   
   2nd Int = \(a+1\)
   
   3rd Int = \(a+2\)
   
   Each integer is one step away from the next on a number line.

2) **Consecutive Odd Integers**
   
   Ex) First Consecutive Odd Integer is \(y\)
   
   Second " " " is \(y+2\)
   
   Third " " " is \(y+4\)
   
   Fourth " " " is \(y+6\)
   
   Each consecutive odd integer is two steps away from the next on a number line.

3) **Consecutive Even Integers**
   
   Ex) First Consecutive Even Integer is \(E\)
   
   2nd " " " is \(E+2\)
   
   3rd " " " is \(E+4\)
   
   4th " " " is \(E+6\)
   
   Each consecutive even integer is two steps away from the next on a number line.

2) **Geometric Area Formulae:**

   **a) Rectangle**  \(A = LW\)
   
   Area = Length times Width

   **b) Square**  \(A = s^2\)
   
   Area = Side times Side

   **c) Triangle**  \(A = \frac{1}{2} \times b \times h\)
   
   Area = One half the product of the base times height

3) **Pythagorean Theorem**
   
   In a right triangle  \(c^2 = a^2 + b^2\)
   
   Longest Side (side opposite the right angle)

   Legs (sides that form RT. \(x\))
In this section, we will look at a variety of word problems that can be solved using quadratic equations. To solve the quadratic equations in this section, we will use the factoring technique presented in Section 13.5. Consider the following examples.

**EXAMPLE 1** Three times the product of two consecutive even integers is 144. Find the integers.

**Solution** The problem asks us to find two integers.

Let \( x \) = first integer

\[
x + 2 = \text{second integer}
\]

Define the variables.

\[
3x(x + 2) = 144
\]

Write an equation relating the variables.

\[
3x^2 + 6x = 144
\]

Rewrite the quadratic equation in standard form.

\[
3x^2 + 6x - 144 = 0
\]

Solve the equation by factoring.

\[
3(x - 6)(x + 8) = 0
\]

\[
x - 6 = 0 \text{ or } x + 8 = 0
\]

\[
x = 6 \text{ or } x = -8
\]

The solution presents us with two sets of answers.

\[
x = 6 \text{ is the first integer } \text{ or } x = -8 \text{ is the first integer}
\]

\[
x + 2 = 8 \text{ is the second integer } \text{ or } x + 2 = -6 \text{ is the second integer}
\]

The two integers are 6 and 8 or -8 and -6.

**Check**

\[
3(6)(8) = 3(48) = 144 \quad \checkmark
\]

\[
3(-8)(-6) = 3(48) = 144 \quad \checkmark
\]

The next three examples involve the use of formulas involving geometric figures. Students who are not familiar with these formulas should refer to Chapter 9, Introduction to Geometry.

**EXAMPLE 2** The length of a rectangle is 5 in. longer than twice its width. If the area of the rectangle is 52 in.\(^2\), what are the dimensions of the rectangle?

**Solution** The problem asks us to find the length and width of a rectangle.

Let \( x \) = width

\[
2x + 5 = \text{length}
\]

\[
A = 52 \text{ in.}^2
\]

The area of a rectangle is length times width.

\[
2x + 5
\]

Substitute into the formula to obtain an equation relating the variables.

\[
52 = (2x + 5)x
\]

Rewrite the quadratic equation in standard form.

\[
52 = 2x^2 + 5x
\]

\[
2x^2 + 5x - 52 = 0
\]

\[
(2x + 13)(x - 4) = 0
\]

Solve the quadratic equation by factoring.

\[
2x + 13 = 0 \text{ or } x - 4 = 0
\]

\[
x = -\frac{13}{2} \text{ or } x = 4
\]

Ignore the solution \( x = -\frac{13}{2} \) since the dimensions must be positive.

\[
x = 4 \text{ in. is the width.}
\]

\[
2x + 5 = 13 \text{ in. is the length.}
\]

The width of the rectangle is 4 in. and its length is 13 in.

**Check**

\[
A = l \cdot w = 13 \cdot 4 = 52 \quad \checkmark
\]
EXAMPLE 3  A 15-foot guy wire runs from the top of a telephone pole to a stake in the ground. If the height of the telephone pole is 3 feet more than the distance along the ground from the base of the pole to the stake, find the height of the telephone pole.

Solution  Draw a diagram to illustrate the problem.

```
       x + 3
       /|
      / |
     /  |
    /   |
   /    |
  /     |
/       |
```

The problem asks you to find the two legs of a right triangle.
Let

\[ x = \text{distance from pole to stake} \]

\[ x + 3 = \text{height of telephone pole} \]

To solve this problem we will use the Pythagorean Theorem.

\[ a^2 + b^2 = c^2 \]

\[ x^2 + (x + 3)^2 = 15^2 \]

\[ x^2 + x^2 + 6x + 9 = 225 \]

\[ 2x^2 + 6x - 216 = 0 \]

\[ 2(x^2 + 3x - 108) = 0 \]

\[ 2(x + 12)(x - 9) = 0 \]

\[ x + 12 = 0 \quad \text{or} \quad x - 9 = 0 \]

\[ x = -12 \quad \text{or} \quad x = 9 \]

Reject the solution \( x = -12 \) since the dimensions must be positive.

\( x = 9 \) ft is the distance from pole to stake.

\( x + 3 = 12 \) ft is the height of the telephone pole.

The telephone pole is 12 ft. high.

Check

\[ 9^2 + 12^2 = 81 + 144 = 225 = (15)^2 \quad \checkmark \]
EXAMPLE 4  The base of a triangle is 4 times its height. If the base is decreased by 2 cm and the height is increased by 5 cm, the area of the new triangle is 90 cm². Find the area of the original triangle.

Solution  The problem asks us to find the area of the original triangle. First we must find its dimensions.

Let  
\[ x = \text{height of original triangle} \]
\[ 4x = \text{base of original triangle} \]

Let  
\[ x + 5 = \text{height of new triangle} \]
\[ 4x - 2 = \text{base of new triangle} \]

The area of a triangle is \( \frac{1}{2} \) base times height.

\[
90 = \frac{1}{2}(4x - 2)(x + 5)
\]

\[
90 = (2x - 1)(x + 5)
\]

\[
90 = 2x^2 + 9x - 95 = 0
\]

\[
(2x + 19)(x - 5) = 0
\]

\[
2x + 19 = 0 \quad \text{or} \quad x - 5 = 0
\]

\[
x = -\frac{19}{2} \quad \text{or} \quad x = 5
\]

Reject the solution \( x = -\frac{19}{2} \) since the dimensions must be positive.

\[ x = 5 \text{ cm is the height of the original triangle.} \]

\[ 4x = 20 \text{ cm is the base of the original triangle.} \]

To find the area of the original triangle we use the formula

\[
A = \frac{1}{2} b \cdot h
\]

\[
A = \frac{1}{2}(20)(5)
\]

\[ A = 50 \text{ cm}^2 \text{ is the area of the original triangle.} \]

The area of the original triangle is 50 cm².

Check  Verify that the area of the new triangle is 90 cm² if \( x = 5 \):

\[
A = \frac{1}{2} b \cdot h = \frac{1}{2}(x + 5)(4x - 2) = \frac{1}{2}(10)(18) = 90 \text{ cm}^2 \quad \checkmark
\]